A model for the diffuse attenuation coefficient of downwelling irradiance

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[1] The diffuse attenuation coefficient for downwelling irradiance (Kd) is an important parameter for ocean studies. For the vast ocean the only feasible means to get fine-scale measurements of Kd is by ocean color remote sensing. At present, values of Kd from remote sensing are estimated using empirical algorithms. Such an approach is insufficient to provide an understanding regarding the variation of Kd and contains large uncertainties in the derived values. In this study a semianalytical model for Kd is developed based on the radiative transfer equation, with values of the model parameters derived from Hydrolight simulations using the averaged particle phase function. The model is further tested with data simulated using significantly different particle phase functions, and the modeled Kd are found matching Hydrolight Kd very well (~2% average error and ~12% maximum error). Such a model provides an improved interpretation about the variation of Kd and a basis to more accurately determine Kd (especially using data from remote sensing).


1. Introduction

[2] Diffuse attenuation coefficient for downwelling irradiance (Kd) (see Table 1 for symbols and definitions used in this text) is an important property for ocean studies. Kd can be used to classify water classes [Jerlov, 1976], and Kd is a critical parameter for accurate estimation of the light intensity at depth [Simpson and Dickey, 1981]. For the vast ocean, satellite remote sensing is the only feasible means to get repetitive and fine-scale measurements of Kd, At present, the standard method to estimate values of Kd from remotely sensed data is through empirical relationships between Kd and the spectral ratio of water-leaving radiance at two wavelengths [Austin and Petzold, 1981; Mueller and Trees, 1997]. Such an approach is insufficient to provide an understanding regarding the variation of Kd, and contains large uncertainties inherent to empirical algorithms [Mueller, 2000]. For the estimation of Kd, which is important for studies of heat budgets [Lewis et al., 1990; Morel and Antoine, 1994; Ohlmann et al., 1996; Zaneveld et al., 1981] and photosynthesis [Marra et al., 1995; Platt et al., 1988; Sathyendranath et al., 1989], a model that can provide better accuracy is desired. In this study, after a brief review of historical descriptions regarding Kd, we developed a semianalytical model based on the radiative transfer equation with model parameters evaluated from Hydrolight numerical simulations. Combined with existing semianalytical algorithms for the derivation of water’s absorption and backscattering coefficients, this Kd model can then be easily implemented to semianalytically calculate values of Kd from remotely sensed data.

2. Background

[3] In ocean optics, spectral Kd at a geometric depth (so-called local value) is defined as [Gordon et al., 1980]

Kd(z) = -1
E_d(z)
\frac{dE_d(z)}{dz},
(1)

with E_d(z) the spectral downwelling irradiance at depth z and z pointing downward from the surface. Wavelength dependence is omitted for brevity. Kd(z) is an apparent optical property (AOP) [Preisendorfer, 1976], which varies with the angular distribution of the light field [Gordon,
1989; Gordon et al., 1975; Kirk, 1984). Since the light distribution changes with depth [Tyler, 1960], \( \textit{K}_d(z) \) varies with \( z \) even for vertically homogenous waters before reaching an asymptotic value at greater depths [Berwald et al., 1995; Liu et al., 2002; McCormick, 1995; McCormick and Hojerslev, 1994; Zaneveld, 1989].

[4] To know the vertical variation of \( \textit{K}_d(z) \), \( Ed(z) \) needs to be measured within a infinitesimal range of \( z \) (see equation (1)). In the field measurements of \( \textit{K}_d \), wave-induced fluctuations in the subsurface light field make it nearly impossible to accurately determine \( \textit{K}_d(z) \). To overcome this obstacle, a common and useful practice is to calculate the diffuse attenuation coefficient between the irradiances measured over distant depths and get

\[
\textit{K}_d(z_1 \leftrightarrow z_2) = \frac{1}{z_2 - z_1} \ln \left( \frac{Ed(z_1)}{Ed(z_2)} \right),
\]

with \( z_1 \) and \( z_2 \) far apart to ensure reliable measurements of \( Ed \) change. In addition, when there are vertical profiles of \( Ed(z) \), \( \textit{K}_d(z_1 \leftrightarrow z_2) \) is usually derived by linear regression analysis between \( \ln(Ed(z)) \) and \( z \) [Darecki and Stramski, 2004; Smith and Baker, 1981]. Clearly, this \( \textit{K}_d(z_1 \leftrightarrow z_2) \) or \( \textit{K}_d \) is not exactly the \( \textit{K}_d(z) \) defined by equation (1). However, this \( \textit{K}_d(z_1 \leftrightarrow z_2) \) is more useful than \( \textit{K}_d(z) \) [McCormick and Hojerslev, 1994] since known \( \textit{K}_d(z_1 \leftrightarrow z_2) \) and \( Ed(z_1) \) (or \( Ed(z_2) \)) makes it easy to calculate \( Ed(z_2) \) (or \( Ed(z_1) \)). For estimating the light intensity at a depth \( z \) [Sathyendranath and Platt, 1988], it is \( \textit{K}_d(z)\), the averaged attenuation coefficient between surface and \( z \) needed, not the local value \( \textit{K}_d(z) \). Also, for measurements made by sensors at fixed depths (e.g., MOBY [Clark et al., 2002]), it is \( \textit{K}_d(z) \) that can be evaluated. Therefore the focus in this study is the variation of \( \textit{K}_d(z) \), not \( \textit{K}_d(z) \). Note that the value represented by symbol \( \textit{K}_d \) in many literatures [McCain et al., 1996; Sathyendranath and Platt, 1988] is actually \( \textit{K}_d(z) \) not the \( \textit{K}_d(z) \) defined by equation (1). Further, the symbol \( \textit{K}_d \) used in this text generally refers to the concept of diffuse attenuation coefficient for \( Ed(z) \), its exact value can be either \( \textit{K}_d(z) \) or \( \textit{K}_d(z) \).

[5] To get \( \textit{K}_d(z) \) value from ocean color remote sensing, one standard method uses empirical spectral ratios [Austin and Petzold, 1981; Mueller and Trees, 1997], another uses pigment concentrations ([C]) [Morel, 1988] with [C] also empirically derived from spectral ratios [Gordon and Morel, 1983; O’Reilly et al., 1998]. Owing to the empirical approaches used in the algorithms, the derived \( \textit{K}_d \) from both methods may contain big uncertainties [Darecki and Stramski, 2004]. Mobley [1994, p. 135] contends that the values of \( \textit{K}_d \) at 450 nm estimated using [C] values can differ by a factor of 2 even for averaged case 1 waters [Morel, 1988]. A 30% error in \( \textit{K}_d \) can result in a factor of 2 error in the calculated \( Ed(E_{10%}) \) where \( Ed(z) \) is 10% of \( Ed(0) \). Therefore a factor of 2 in \( \textit{K}_d \) will lead to significantly wrong \( Ed(E_{10%}) \) value.

[6] To understand the nature of \( \textit{K}_d \) variations, efforts have been made to link \( \textit{K}_d(z) \) with water’s inherent optical properties (IOPs) [Preisendorfer, 1976], such as absorption (\( a \)), scattering, (\( b \)), and or backscattering (\( \mu_b \)) coefficients. For instance, through Monte Carlo simulations, Gordon [1989] empirically approximated \( \textit{K}_d(0) \) as 1.04(\( a + b_b \))/\( \mu_0 \) with \( \mu_0 \) the average cosine of the downwelling light just beneath the surface. Kirk [1984, 1991] and Morel and Loisel [1998] empirically modeled \( \textit{K}_d \) of the euphotic zone ((\( \textit{K}_d(z) \)) as a function of \( a \) and \( b \) with the formula

\[
\frac{\langle \textit{K}_d(z) \rangle}{a} = \frac{1}{\mu_0} 
\left(1 + G(\mu_0) \frac{b_b}{a} \right)^{0.4}.
\]

\( G(\mu_0) \) is a model parameter that determines the relative contribution of scattering to \( \langle \textit{K}_d(z) \rangle \). Values of \( G(\mu_0) \) are found to vary with both \( \mu_0 \) and the volume scattering function (VSF) of the water medium [Kirk, 1991; Morel and Loisel, 1998]. For \( \textit{K}_d \) of the upper half of the euphotic zone \( \textit{K}_d \) of the asymptotic value, they are also empirically expressed functions of \( a \) and \( b \) in earlier studies [Gordon, 1989; Gordon et al., 1975; Zaneveld, 1989].

[7] VSF (or scattering phase function of particles) is a property that is seldom measured in the field and cannot be analytically derived from ocean color remote sensing. Gordon [1993] has found that below-surface upwelling radiance is not sensitive to change of \( b \) for a given \( b_b \). Therefore values of \( b \) can hardly be accurately derived from ocean color remote sensing, because only signals of upwelling radiance are collected by a remote sensor. These limitations undermine the application of models.

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Table 1. Symbols and Definitions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
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<tbody>
<tr>
<td>( a )</td>
<td>absorption coefficient</td>
<td>m(^{-1} )</td>
</tr>
<tr>
<td>( b )</td>
<td>scattering coefficient</td>
<td>m(^{-1} )</td>
</tr>
<tr>
<td>( b_b )</td>
<td>backscattering coefficient</td>
<td>m(^{-1} )</td>
</tr>
<tr>
<td>( Ed(z_1) )</td>
<td>downwelling (upwelling) irradiance at depth ( z )</td>
<td>W m(^{-2} ) nm(^{-1} )</td>
</tr>
<tr>
<td>( Ed(z_2) )</td>
<td>scalar irradiance at depth ( z )</td>
<td>W m(^{-2} ) nm(^{-1} )</td>
</tr>
<tr>
<td>( R(z) )</td>
<td>remote sensing reflectance (ratio of water-leaving radiance to downwelling irradiance above the surface)</td>
<td>sr(^{-1} )</td>
</tr>
<tr>
<td>( K_d )</td>
<td>diffuse attenuation coefficient for downwelling irradiance</td>
<td>m(^{-1} )</td>
</tr>
<tr>
<td>( K_d(z) )</td>
<td>diffuse attenuation coefficient for downwelling irradiance at depth ( z ), the so-called local value</td>
<td>m(^{-1} )</td>
</tr>
<tr>
<td>( K_d(z_{eup}) )</td>
<td>( K_d(z) ) of the euphotic zone (between ( E_d(0) ) and 1% of ( E_d(0) ))</td>
<td>m(^{-1} )</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>diffuse attenuation coefficient for downwelling irradiance between 0 m and depth ( z )</td>
<td>m(^{-1} )</td>
</tr>
<tr>
<td>( \mu_0(z) )</td>
<td>average cosine of downwelling (upwelling) light at depth ( z )</td>
<td>deg</td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td>average cosine of subsurface downwelling light</td>
<td>deg</td>
</tr>
</tbody>
</table>

*(Note that except \( \theta_0 \), all other properties more or less vary with wavelength.)*
like equation (3) to estimate \( K_d \) (especially for ocean color remote sensing).

[8] On the other hand, it has been demonstrated that absorption \( (a) \) and backscattering \( (b) \) coefficients can be well retrieved from ocean color remote sensing [Garver and Siegel, 1997; Hoge and Lyon, 1996; Lee et al., 2002, 1996; Loisel and Stramski, 2000; Roesler and Boss, 2003; Roesler and Perry, 1995]. The accuracy for \( a \) and \( b \) derived from remote-sensing reflectance can be within \( \sim 15\% \) [Hoge and Lyon, 1996; Lee et al., 2002; Loisel and Stramski, 2000; Loisel et al., 2001], in contrast to the factor of 2 error in derived [C] [Gordon and Morel, 1983; O’Reilly et al., 1998]. Therefore a more accurate approach is to evaluate \( K_d(z) \).

Here \( K_d(z) \) is often simplified as [Smith and Baker, 1981]

\[
K_d = a + b_v
\]

or [Sathyendranath and Platt, 1988; Sathyendranath et al., 1989]

\[
K_d = (a + b_v)/\mu_v.
\]

These approximations, however, were not elaborated from radiative transfer. To establish a theoretical basis and to improve the determination of \( K_d(z) \), it is necessary to refine these simple approximations based on the radiative transfer equation (RTE).

3. General Form to Model \( K_d(z) \)

[9] After integrating the RTE for the upward and downward light field, there is [Aas, 1987; Stavn and Weidemann, 1989]

\[
\frac{dE_d(z)}{dz} = -\frac{a}{\mu_v(z)}E_d(z) - \frac{r_s(z) b_v}{\mu_v(z)} E_d(z) + \frac{r_s(z) b_v}{\mu_v(z)} E_u(z).
\]

Here \( \mu_v(z) \) is the average cosine and \( r_s(z) \) are the shape factors for downwelling (upwelling) light field [Stavn and Weidemann, 1989], respectively. \( E_u \) is the upwelling irradiance. Apply the definition of \( R \) (ratio of \( E_u \) to \( E_d \) [Gordon et al., 1980]) and the definition of \( K_d(z) \) (equation (1)), there is

\[
K_d(z) = \frac{1}{\mu_v(z)} a + \left( \frac{r_s(z)}{\mu_v(z)} - \frac{r_s(z) R(z)}{\mu_v(z)} \right) b_v.
\]

As \( K_d(z) \) is the average of \( K_d \) between 0 m and \( z \), \( K_d(z) \) can be expressed as

\[
K_d(z) = m_0(z) a + \nu(z) b_v.
\]

[10] Equation (7) is simply a rewrite of equation (6) by introducing two parameters \( m_0 \) and \( \nu \) to represent the combined effects of \( \mu_v(z) \) and \( r_s(z) \). This RTE-based expression reveals an important concept that has often been overlooked: theoretically the rates of contributions from \( a \) and \( b_v \) to \( K_d(z) \) (or \( K_d \)) are not the same, i.e., \( m_0 \neq \nu \), as opposed to the simplifications of equation (4). Further, when the distribution of the field changes, \( m_0 \) and \( \nu \) may change accordingly.

[11] Since our objective is to obtain an \( a \& b \)-based \( K_d(z) \) model that can be easily used for ocean color studies, the individual variations of \( m_0(z) \) or \( \nu(z) \) are not interested in here. Some analyses and discussions regarding those parameters can be found in the work of Kirk [1981], Stavn and Weidemann [1989], and Ackleson et al. [1994]. Our objective is the values and variations of \( m_0 \) and \( \nu \). When \( m_0 \) and \( \nu \) are known, it is straightforward to calculate \( K_d(z) \) with known values of \( a \) and \( b_v \). The values and variations of \( m_0 \) and \( \nu \), however, are not directly derivable from the RTE or equation (6). As in earlier studies in developing semianalytical models for other apparent optical properties [Lee et al., 2004; Morel and Loisel, 1998], the values and variations of \( m_0 \) and \( \nu \) are derived and analyzed from numerical simulations of the RTE.

4. Hydrolight Simulations

[12] As many studies [Berwald et al., 1995; Lee et al., 1998; Mobley et al., 1993, 2002; Morel and Loisel, 1998], we used the widely accepted Hydrolight [Mobley, 1995] to simulate the subsurface light field. Values of \( K_d(z) \) were then calculated from the simulated \( E_d(z) \) by equation (2).

[13] For Hydrolight simulations, the input data are solar light and water’s IOPs. As those earlier studies, the downwelling irradiance at sea surface from the Sun and sky is simulated by the spectral model of Gregg and Carder [1990]. Wind speeds of 5 m/s is assigned, and a series of water depths are selected. The required IOPs are the absorption and scattering coefficients and the particle phase function (PPF). As earlier studies [Gordon, 1989; Kirk, 1991; Morel and Loisel, 1998], these IOPs are kept vertically constant. Also, the scattering is separated into molecular and nonmolecular (collectively called particle) scatterings [Gordon and Morel, 1983; Morel and Gentili, 1993; Sathyendranath et al., 2001]. The total absorption (\( a \)) and backscattering (\( b_v \)) coefficients are taken from the data set adopted by the International Ocean-Color Coordinating Group (see http://www.oiocg.org/groups/OCAG_data.html), which has 500 different spectra (400–800 nm with a step of every 10 nm) of \( a \) and \( b_v \) that cover wide dynamic ranges. Three realistic but significantly different PPFs are employed to provide different values of \( b \) for each \( b_v \). One of the PPFs is the averaged particle phase function (AVGP) from Petzold’s measurements [Mobley et al., 2002; Petzold, 1972], which has a backscattering to total-scattering ratio (\( a = b_v/\sigma \)) of 1.83%. The other two PPFs are simulated by the Fournier-Forand model [Fournier and Forand, 1994; Mobley et al., 2002], with \( \sigma \) values as 1.0% (represented as FF010) and 4.0% (represented as FF040), respectively. Therefore for each \( b_v \) value, there are three significantly different \( b \) values in the numerical simulations.

[14] To reduce the calculation time but without losing representation of the dynamic range of the data set, \( a \) and \( b_v \) spectra were selected from the first data point with a step of 5. Therefore 100 pairs of \( a \) and \( b_v \) spectra (\( a(440) \) ranged from 0.016 to 3.1 m\(^{-1} \) and \( b_v(440) \) ranged from...
0.0034 to 0.113 m$^{-1}$ are used as IOP inputs for the Hydrolight simulations.

5. Results

5.1. Some Characteristics

As an example to show the variation of $K_d(z)$, values of the diffuse attenuation coefficient between 0 m and 1 m ($K_d(1)$, AVGP as PPF) are presented here, since similar results are found for other $K_d(z)$. For data shown in Figure 1, the Sun is at 30° from zenith. Following Kirk [1984], Figure 1a plots the values of $K_d(1)/a$ against the values of $b/a$, with Figure 1b showing $K_d(1)/a$ against the values of $b_b/a$. Not surprisingly, as those shown by Kirk [1984] and Morel and Loisel [1998], $K_d(1)/a$ increases with $b/a$ due to increased scattering. $K_d(1)/a$ spans a range of 1.1 to 2.2 for the different combinations of IOPs, and scatters about ±12% around its average value for a corresponding $b/a$ (or $b_b/a$) value. The deviations of these $K_d(1)/a$ values are wider compared to the $K_d(E_{10%})/a$ shown by Kirk [1984]. This is because $K_d(E_{10%})$ is a local value and is corresponding to a fixed ratio (10%) of $E_d(z)/E_d(0)$. The results in Figure 1a, however, are for fixed depth range (0–1 m) and have $E_d(1)/E_d(0)$ ranging from 98 to 0.2% due to the wide variations of IOPs. Since the light field at 1 m is different for the different IOPs, and $K_d$ is a property depending on the distribution of light field, we see $K_d(1)/a$ not a constant for the same $b/a$.

To see how $K_d(1)$ varies with $a$ and $b_b$, Figure 2a shows the variation of $K_d(1)/a$ versus $b_b/a$ for a series of $a$ values, while Figure 2b shows the variation of $K_d(1)/a$ versus $a$ for different $b_b/a$ values. Apparently, for data shown in Figure 2a (total absorption coefficients varied from 0.03 to 1.0 m$^{-1}$), $K_d(1)/a$ follow linear relationships with $b_b/a$ (correlation coefficient ($r^2$) > 0.99). On the other hand, for the different $b_b/a$ values (Figure 2b), $K_d(1)/a$ increases with $a$, but in a nonlinear fashion and apparently approaching an asymptotic value. This phenomenon is consistent with the asymptotic theory [McCormick, 1992; Zaneveld, 1989] that eventually $K_d(z)$ approaches an asymptotic value at larger optical depths. Here the range of geometric depth is fixed between 0 and 1 m, but the increase of $a$ and $b_b$ will increase the optical depths.

Figure 1. Hydrolight-simulated $K_d(1)/a$ versus inherent optical properties (IOPs) (4100 points), with the Sun at 30° from zenith and particles following the averaged particle phase function (AVGP). (a) $K_d(1)/a$ versus $b/a$. (b) $K_d(1)/a$ versus $b_b/a$.

Figure 2. $K_d(1)/a$ (a) versus $b_b/a$ for different $a$ values (in the box) and (b) versus $a$ for different $b_b/a$ values (in the box). All data are from Figure 1.
For data shown in Figure 2a, the intercepts from linear regression analysis between $K_d(1)/a$ and $b_d/a$ are about the same (~1.12) for all absorption coefficients. The slope (value of $v$), however, differs a lot for various absorption coefficients. The $v$ values range from 1.67 for $a = 0.03$ m$^{-1}$ to 3.79 for $a = 1.0$ m$^{-1}$. These results indicate that (1) for different light fields (resulted from different $a$ and $b_d$ values), the contribution rate (value of $v$) of $b_d$ to $K_d(1)$ differs, as indicated by equation (7) and (2) the contribution rate of $b_d$ to $K_d(1)$ can be three times the rate of $a$ to $K_d(1)$. These results indicate that the simple $K_d$ model (equations (4a) or (4b)) underestimates the contributions of $b_d$ to $K_d$. For particle free waters, Gordon [1989] found that the Sun angle-normalized $K_d$ is about $a + 1.44b_d$. Apparently our results are consistent with that of Gordon [1989] for low-absorption waters. Note that the absorption coefficients of clearest natural water in the work of Smith and Baker [1981] were derived from $K_d$ using equation (4a), and their values in the blue wavelengths are found to be larger than those measured by Pope and Fry [1997]. The actual dependence of $K_d$ on $a$ and $b_d$ suggests that at least a portion of the difference can be removed if a more accurate $K_d$ model is used in the derivation [Gordon, 1989].

5.2. $K_d(1)$ for Different Particle Phase Functions

The above only shows the variation of $K_d(1)/a$ for one PPF; it is desired and important to know how $K_d(1)/a$ varies with different particle phase functions (PPFs). As described earlier, Hydrolight simulations with three different PPFs were carried out. In these simulations, it was the same set of absorption and backscattering coefficients used. Change of PPF only changes the total scattering coefficient for each $b_d$. For example, Figure 3a shows $K_d(1)/a$ versus $b/a$ for the three different PPFs with the absorption coefficient set at 0.3 m$^{-1}$. Clearly, as shown by Kirk [1991] and Morel and Loisel [1998], $K_d(E_{VGP})$, $K_d$, and $K_d(1)$ vary significantly for different PPFs even $a$ and $b/a$ values are kept the same. These results further emphasis that knowing $a$ and $b$ is not enough for the estimation of $K_d$, and it is critical to know the PPF (or VSF) of the water in order to get accurate $K_d$ when using models like equation (3).

Figure 3b shows the relationship between $K_d(1)/a$ and $b_d/a$ for the data used in Figure 3a. Apparently, among the three realistic PPFs, the dependence of $K_d(1)/a$ on $b_d/a$ is much more stable than that of $K_d(1)/a$ on $b/a$, though it is not clear why no apparent difference exists between the results using AVGP as PPF and that using FF010 as PPF. For the three PPFs with $\sigma = b_d/b$ values varied by a factor of 4 (1.0% to 4.0%), the $K_d(1)/a$ values deviate ±5% around its average for a $b_d/a$ value. These results suggest that the contribution rates of $a$ and $b_d$ to $K_d(1)$ for given $a$ and $b_d$ do not vary much for different $b$ values. This is consistent with equation (7), and demonstrates that it is the absorption and backscattering coefficients contributing most to $K_d$. The forward scattering coefficient has only secondary effects on $K_d$. This result indicated that if $a$ and $b_d$ (along with other auxiliary information, such as solar altitude) are known, $K_d(2)$ can be adequately calculated without exact knowledge of the particle phase function, as indicated in earlier studies [Sathyendranath and Platt, 1988; Smith and Baker, 1981]. This is important for ocean color remote sensing, as it can best provide the absorption and backscattering coefficients, not the PPF or VSF or $b$ [Gordon, 1993].

5.3. Model $K_d(2)$ as a Function of $a$ and $b_d$

To make the $K_d(2)$ model (equation 7) useful for ocean studies, how values of $m_0$ and $v$ vary with water properties and solar zenith angle needs to be known. Figure 2a indicates that for a specified Sun angle $m_0$ is nearly a constant (~1.12), whereas $v$ changes in a bigger range (1.67 to 3.79, for instance). Further, since data shown in Figure 2b suggests that $v$ increases nonlinearly with $a$ and reaches an asymptotic value for large $a$, slope $v$ (which is also the contribution rate of $b_d$ to $K_d$) is empirically modeled as follows:

$$v = m_1(1 - m_2e^{-m_3a}).$$  \hspace{1cm} (8)

Combining equation (7) with equation (8), there is

$$K_d(1) = m_0a + m_1(1 - m_2e^{-m_3a})b_d.$$ \hspace{1cm} (9)

In this semianalytical relationship between $K_d(1)$ and $a$ and $b_d$, four model parameters ($m_0$, $m_1$, $m_2$, and $m_3$) are employed to
explicitly quantify the contribution rates of $a$ and $b_n$ to $K_d(1)$. Following the method described in many earlier studies [Gordon, 1989; Kirk, 1991; Morel and Loisel, 1998], the values of these model parameters (provided in Table 2) are derived by curve fitting all data points shown in Figure 1b with equation (9). Figure 4 compares the $K_d(1)$ values modeled by equation (9) versus those calculated from Hydrolight simulations. As expected, excellent model results were obtained for these $K_d(1)$ values, which span a range of three orders of magnitude (0.025 – 6.1 m$^{-1}$). The average error is less than 1% for all 4100 points, with a maximum error about 2%. Note that the model parameters are the same for all $a$ and $b_n$, thus the $K_d(1)$ model is independent of the biogeochemical-to-optical relationships, as long as values of $a$ and $b_n$ are provided.

[21] The above results show $K_d(1)$ with the Sun at 30°. For studies in physical and biological oceanography [Sathyendranath and Platt, 1988] and for measurements made at a series of fixed depths (MOBY or HyCODE mooring [Chang and Dickey, 2004], for instance), it is desired to have a system that provides $K_d(z)$ for other depths and Sun angles. On the basis of the results of $K_d(1)$, a generalized model for $K_d(z)$ is expressed as

$$K_d(z, 0) = m_0(z, 0) a + m_1(z, 0) \left(1 - m_2(z, 0) e^{-m_3(z, 0)w}\right) b_n.$$  

(10)

In this model, the values of the four model parameters ($m_0$, $m_1$, $m_2$, $m_3$) are needed for desired solar altitude and depth. This requirement is fulfilled by a look-up table (LUT) [Liu et al., 2002; Morel and Gentili, 1993]. For a few depths and solar altitudes, the derived model parameters ($m_0$, $m_1$, $m_2$, $m_3$) are provided in Table 2. Owing to the change of light distribution, values of $m_0$, $m_1$, $m_2$, and $m_3$ vary more or less with depth and solar altitude. Note that these values were derived with data satisfying $E_d(z)/E_d(0) > 10^{-5}$ simply because the $E_d(z)$ values that below $10^{-5}$ of $E_d(0)$ cannot be well measured in the field and make a negligible contribution to photosynthesis. For these $K_d(z)$ data, the average error is $\sim 3\%$ (maximum error $\sim 10\%$) between equation (10) modeled $K_d(z)$ and Hydrolight determined $K_d(z)$. For example, with the AVGP as PPF for particle scattering, Figure 5 shows modeled $K_d(5)$ versus Hydrolight $K_d(5)$ for the Sun at 10° and 60° from zenith, respectively.

### 5.4. $K_d$ for the Euphotic Zone ($K_d(E_{10\%})$)

[22] To simplify the process of calculating $E_d(z)$ of the surface layer and accept coarser approximations in calculated $E_d(z)$ [Sathyendranath and Platt, 1988], $K_d(z)$ in the euphotic zone might be replaced by the value of $K_d(E_{10\%})$, the $K_d$ value between $E_d(0)$ and 10% of $E_d(0)$. Practically, this is the layer that contributes most to the photosynthesis of the water column [Antoine et al., 1995; Platt, 1986]. Also, most signals measured by a remote sensor are originated in this surface layer [Gordon and Mclune, 1975], and many field-measured $K_d$ are derived from measurements made in this layer. As with the above model of $K_d(z)$, values of $m_0$, $m_1$, $m_2$, and $m_3$ are derived for $K_d(E_{10\%})$ from Hydrolight simulations and are provided (last row of Table 2). To include a wider range of $K_d(E_{10\%})$ needed for better derivation of the ($m_0$, $m_1$, $m_2$, $m_3$) values, the $K_d(E_{10\%})$ values from Hydrolight simulations were actually for $E_d(z)/E_d(0)$ between 8% and 12%, instead of the exact 10%. For these $K_d(E_{10\%})$ values, the average error is $\sim 2\%$ (maximum error $\sim 9\%$) between modeled and Hydrolight $K_d(E_{10\%})$. As an example, Figure 6 presents modeled $K_d(E_{10\%})$ versus Hydrolight $K_d(E_{10\%})$ for the Sun at 30°. The data gap for $K_d(E_{10\%})$ around 0.08 m$^{-1}$ is due to the depth selection in the Hydrolight simulations resulting in no $K_d(E_{10\%})$ around 0.08 m$^{-1}$. This data gap, however, has no impact on the application of the model as the agreement between modeled and known $K_d(E_{10\%})$ is good for $K_d(E_{10\%})$ in the range of $\sim 0.05$ – 5.7 m$^{-1}$.

[23] Further, we found that for the four parameters the variations of $m_1$, $m_2$, and $m_3$ are limited for different solar altitude, while most variations happened in $m_0$ (see last row of Table 2). The variation of $m_0$ is conceptually consistent with the results shown by Kirk [1984] and Gordon [1989]. Since there is not much variation for $m_1$, $m_2$, and $m_3$, a set of

### Table 2. Model Parameters for a Few Solar Altitudes and Depth Ranges

<table>
<thead>
<tr>
<th>Range</th>
<th>10°</th>
<th>30°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 1 m</td>
<td>1.060</td>
<td>4.307</td>
<td>0.675</td>
</tr>
<tr>
<td>0 – 5 m</td>
<td>1.048</td>
<td>5.573</td>
<td>0.727</td>
</tr>
<tr>
<td>0 – 10 m</td>
<td>1.094</td>
<td>4.148</td>
<td>0.685</td>
</tr>
<tr>
<td>0 – 20 m</td>
<td>1.049</td>
<td>4.877</td>
<td>0.786</td>
</tr>
<tr>
<td>$K_d(E_{10%})$</td>
<td>1.044</td>
<td>4.173</td>
<td>0.530</td>
</tr>
</tbody>
</table>
averages for these parameters can be derived without losing much accuracy to modeled $K_d(E_{10\%})$. For the variation of $m_0$ with solar altitude, extra Hydrolight simulations with the Sun at 20°, 40°, and 50° were carried out for the same IOPs. On the basis of all Hydrolight simulations, it is found that a generalized model for $K_d(E_{10\%})$ can be

$$K_d(E_{10\%}) = (1 + 0.0050 a + 4.18 (1 - 0.52 e^{-10.8 a}) b_b). \quad (11)$$

Here $\theta_0$ is the above surface solar zenith angle in degrees, such as 30° for instance. To simplify equation (11) to a concise form similar as those in earlier studies [Gordon, 1989; Sathyendranath and Platt, 1988] but with potentially larger errors followed, there is

$$K_d(E_{10\%}) = (1 + 0.0050 a + 3.47 b_b). \quad (12)$$

With equation (11) or equation (12), values of $K_d(E_{10\%})$ can then be quickly determined when values of $\theta_0$, $a$, and $b_b$ are given.

5.5. Test the $K_d(E_{10\%})$ Model With Data Simulated by Other PPFs

[23] The above presented results of $K_d(E_{10\%})$ for one particle phase function. It is equally or more important to know the performance of this model to $K_d(E_{10\%})$ values resulted from other phase functions. For this need, the above $K_d(E_{10\%}) \leftrightarrow a \& b_b$ model (equation (11)) is applied to the Hydrolight data simulated with the other two PPFs without any change in model parameters. Figure 7a shows the $K_d(E_{10\%})$ comparison for FF010 and Figure 7b for FF040, where all data have the Sun at 60° from zenith. The average error is 2.1% (maximum error is 8.8%) for FF010, while the average error is 1.7% (maximum error is 11.7%) for FF040. Clearly, the model performed very well to both data sets. These kinds of results indicate that $K_d(E_{10\%})$ values determined by equation (11) are reliable, though the PPF differed significantly (resulting in a factor of 4 difference in $b$ values for each $b_b$). Like the results of $K_d(1)$, the change of forward-scattering coefficients only showed insignificant effects on $K_d(E_{10\%})$.

6. Discussion

[25] For studies of heat transfer and photosynthesis in the ocean, it requires to know the scalar irradiance at depth ($E_0(z)$) [Morel, 1978]. Fundamentally, $E_0(z)$ can be expressed as

$$E_0(z) = \frac{E_d(z)}{\mu_d(z)} + \frac{E_u(z)}{\mu_u(z)} = \left(\frac{1}{\mu_d(z)} + \frac{R(z)}{\mu_u(z)}\right) E_u(0) e^{-K_d(z) z}. \quad (13)$$

In equation (13), $\mu_d(z)$ is generally in a range of 0.7–0.96 and $\mu_u(z)$ in a range (0.4–0.5) for all depths [Berwald et al., 1995; Kirk, 1981; Mobley, 1994, p. 552], and $R$ is generally less than 0.1 [Kirk, 1991; Morel and Maritorena, 2001; Morel and Prieur, 1977]. Thus it is clear that errors in $E_0$ at any depth resulted from uncertainties in $\mu_d(z)$ and $\mu_u(z)$ are quite limited. The critical parameter in the determination of $E_0$ at depth is $K_d(z)$ (see equation (13)). For instance, a 30% error in $K_d(z)$ can result in a factor of 2 error in $E_0(E_{10\%})$. Thus it is essential to get...
accurate values of $K_d(z)$, though better estimations of $m_d(z)$ and $m_u(z)$ will improve the evaluation of $E_0(z)$. Apparently a model like equation (10) or equation (11) for the diffuse attenuation coefficient of $E_0(z)$ can also be developed. This is omitted here in order to keep this study focused, also because (1) historically and currently values of $K_d(z)$ are extensively measured in the field and (2) $E_0$ at depth can be well determined by equation (13) if the two important parameters ($E_0(0)$ and $K_d(z)$) are known. For higher precision of $E_0(z)$ estimation, models regarding the variation of $m_d(z)$ and $m_u(z)$ [Liu et al., 2002; McCormick, 1995; Zaneveld, 1989] could be adopted.

[26] At this point, a semianalytical model for the diffuse attenuation coefficient ($K_d(z)$) is developed for vertically homogeneous and optically deep water. It is noticed that this semianalytically determined $K_d(z)$ will not be as accurate as that determined from Hydrolight given the same IOPs and boundary conditions, but the explicit algebraic expression developed here processes data almost instantaneously for given IOPs, depths, and solar altitudes. This effectiveness is important for analyzing data from the vast ocean. Further, this model for $K_d(z)$ along with the model for remote-sensing reflectance ($R_{rs}$) compose an explicit system for modeling and processing ocean color data. In this system (schematically depicted in Figure 8), there are three different links to relate the environmental properties, IOPs, and AOPs. Link I relates biogeochemical properties with IOP; Link II relates IOP with $R_{rs}$ or irradiance reflectance ($R$); and Link III relates IOP with $K_d$. Links II and III are bridged by optical-to-optical properties, their models in general have fewer uncertainties when applied to a wide range of waters. Most of the uncertainties occur at Link I, where the conversion factors (e.g., pigment-specific absorption coefficient [Bricaud et al., 1995; Sathyendranath et al., 1987] or biogeochemical-specific scattering coefficient [Gordon and Morel, 1983; Loisel and Morel, 1998]) involved in the biogeochemical-to-optical relationships vary significantly over different regions and/or seasons, therefore regional/temporal relationships have to be adopted to cope with such variations. This is the link that needs increased efforts for the development of regional/temporal algorithms for the concentrations of those biogeochemical constituents.

[27] Since $a$ and $b$, are at the center of the modeling/processing system, and methods/algorithms have been developed to retrieve $a$ and $b$ from $R_{rs}$ [Carder et al., 1999; Hoge and Lyon, 1996; Lee et al., 2002; Loisel and Stramski, 2000; Roesler and Perry, 1995], estimation of $K_d(z)$ from remote sensing can now be carried out in a semianalytical fashion (Link III) that is independent of the derivation of the concentrations (such as $[C]$). Such an approach then avoids the large uncertainties associated with the $[C]$ estimation [O’Reilly et al., 1998], and provides a potential to signifi-

Figure 7. Comparison of $\bar{K}_d(E_{10\%})$ from Hydrolight simulations versus $\bar{K}_d(E_{10\%})$ determined by equation (11), a model developed using data with AVGP as particle phase function (PPF). (a) For Hydrolight data with FF010 as PPF. (b) For Hydrolight data with FF040 as PPF (see text for details).

Figure 8. Structural relationships among biogeochemical concentrations, IOPs, $R_{rs}$ (and/or $R$), and $K_d$. 
cantly improve the determination of $K_d(z)$ from ocean color remote sensing.

7. Conclusions

[28] On the basis of the radiative transfer equation and Hydrolight numerical simulations, a semianalytical model is developed for the diffuse attenuation coefficient of downwelling irradiance ($K_d(z)$). As in some earlier studies [Sathyendranath and Platt, 1988; Smith and Baker, 1981], the model expresses $K_d(z)$ as a function of absorption and backscattering coefficients, but it refined the earlier simple approximations so that the model is now consistent with the theory of radiative transfer and improves the determination of $K_d(z)$. On the basis of extensive numerical simulations, the values of the model parameters are derived by fitting the data with the model. The model parameters vary with solar altitude and depth, but remain the same for different IOPs. At least for data used in this study, this $K_d(z) \leq a \& k_b$ model can be used for data with realistic volume scattering function (or particle phase function). Since $a \& k_b$ can be well retrieved from ocean color remote sensing, the $K_d(z)$ model developed here provides a route to quickly and semianalytically determine its values for pixels of the ocean. Also, for downwelling irradiances that are measured at a few fixed depths (such as MOBY, or TSRB of Satalantic, Inc., etc.), the model developed here provides a basis for the derivation of $a \& k_b$ from measured $K_d(z)$.

[29] Compared to the $K_d(z)$ from exact numerical simulations such as Hydrolight, there are a few percent of errors remaining in the semianalytically modeled $K_d(z)$. Such types of errors are common in semianalytical approaches [Gordon, 1989; Kirk, 1991; Sathyendranath and Platt, 1997]. A semianalytical model, however, provides (1) an explicit way to easily understand the fundamental relationships between IOPs and AOPs [Gordon et al., 1975; Kirk, 1991]; (2) a tool to instantaneously evaluate AOPs for given IOPs; and (3) a basis to quickly invert IOPs from AOPs [Hoge and Lyon, 1996; Lee et al., 2002; Loisel and Stramski, 2000]. At present, calculation efficiency is important for studies of the vast ocean, and a few percent of error is much better than acceptable limits in remote sensing and uncertainties in field measurements. Therefore semianalytical models practically meet both efficiency and accuracy requirements needed for oceanographic studies.

[30] The values of the model parameters are derived with Hydrolight simulations under clear sky conditions and a wind speed of 5 m/s. Adjustments of those values are expected for cloudy conditions and high wind speed. Also, it is necessary to point out that the model developed here and the models of earlier studies [Gordon, 1989; Gordon et al., 1975; Kirk, 1991; Morel and Loisel, 1998] are for vertically homogeneous waters, which are generally applicable for the surface mixed layer of the ocean [Mueller and Lange, 1989]. However, challenges remain to extend the models to vertically stratified waters, especially to remote sensing of the ocean where the extent of stratification of the surface layer is unknown when a satellite sensor makes measurements.

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