Regulation of Enzyme Activity

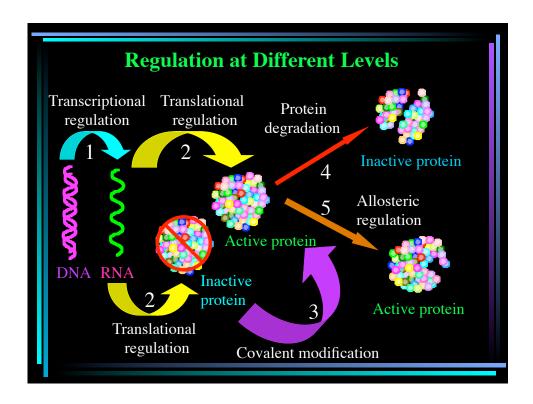
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The Theme of This Lecture

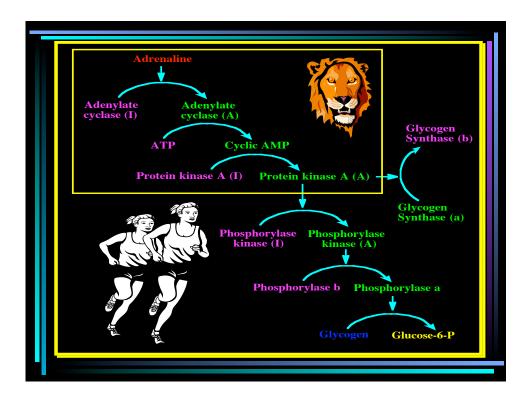
Regulation of Enzyme Activity at Protein Level.

- 1. Covalent modification.
- 2. Noncovalent (allosteric) regulation
- 3. Protein degradation (will not be considered).



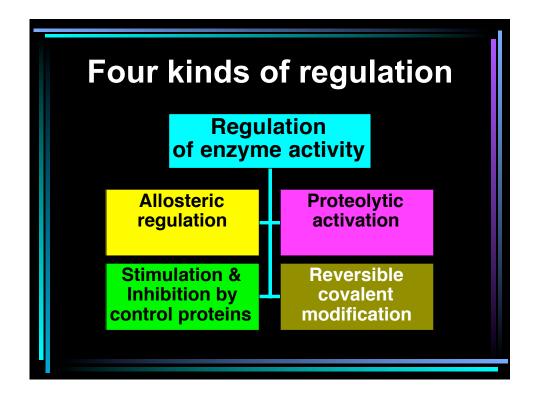
Why regulate?

- Emergency situation Response should be rapid.
- Resource availability should adjust to availability of resources from both internal and external sources.



Amplification of enzyme activity

- Approximately 0.1 nmoles of adrenaline per gram of muscle will trigger the synthesis of 25 μmoles of glucose -1phosphate per minute per gram of muscle.
- This represents an amplification of 250,000 fold.



Proteolytic activation

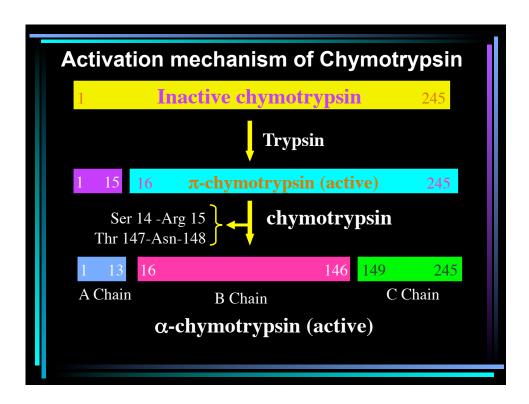
This kind of activation is irreversible.

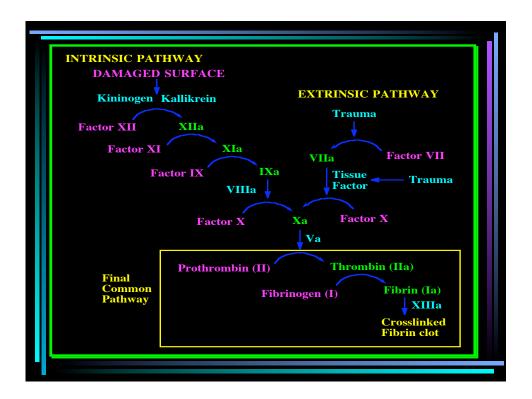
Once the protein is activated, the process cannot be reversed.

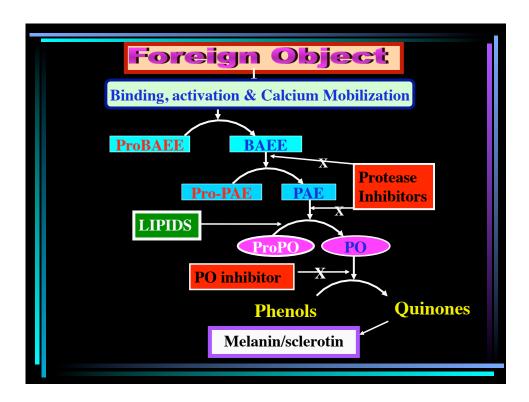
Active protein can only be controlled by other kinds of regulation.

Examples of Proteolytic Activation

- Zymogen activation.
- Blood clotting.
- Complement activation.
- Prophenoloxidase activation.
- Inactive hormones to active hormones.







Reversible Covalent Modification

A single trigger rapidly switches a whole pathway on or off

Examples of reversible covalent modifications

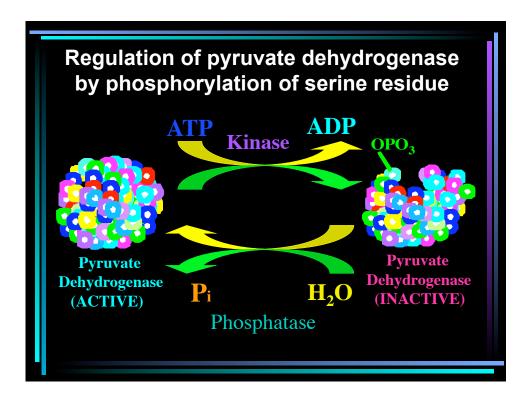
- Phosphorylation dephosphorylation
- Adenylation deadenylation.
- Uridinylation -deuridinylation.
- · Thiol disulfide exchange.
- Methylation demethylation.
- Acetylation -deacetylation.

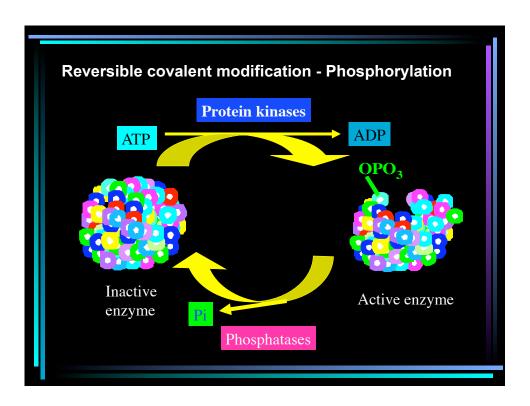
Reversible covalent modification - Phosphorylation

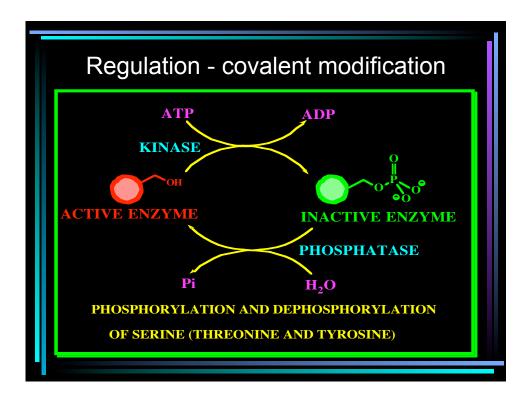
While running, glycogen phosphorylase activity

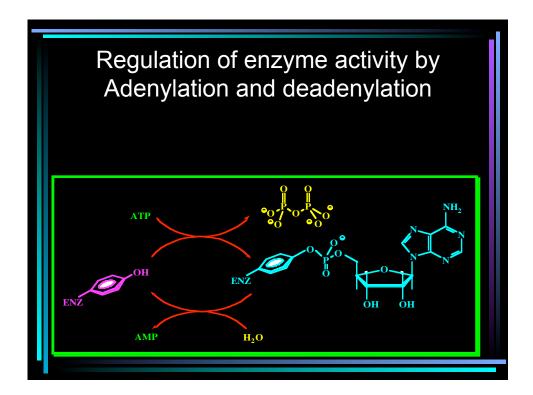
is enhanced by phosphorylation.

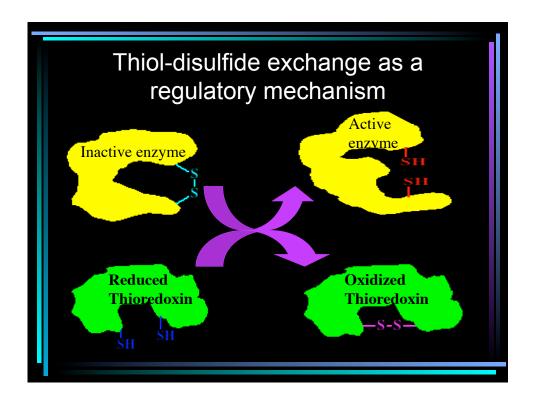
At the same time glycogen synthase activity is shut off by phosphorylation.











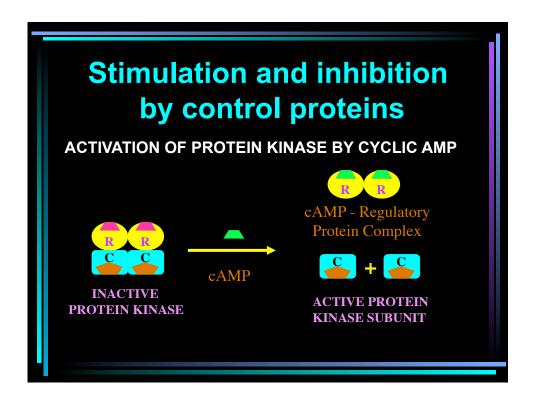
Enzymes regulated by reduction (in plants)

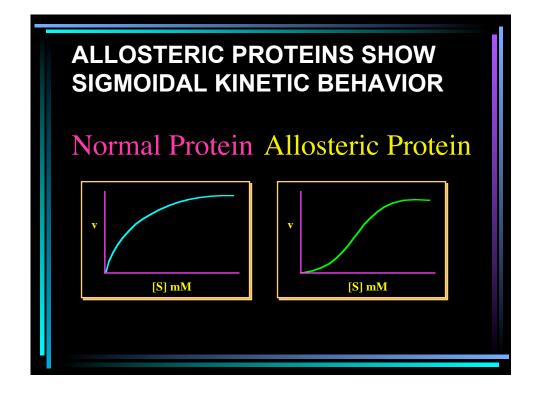
Activation by disulfide reduction

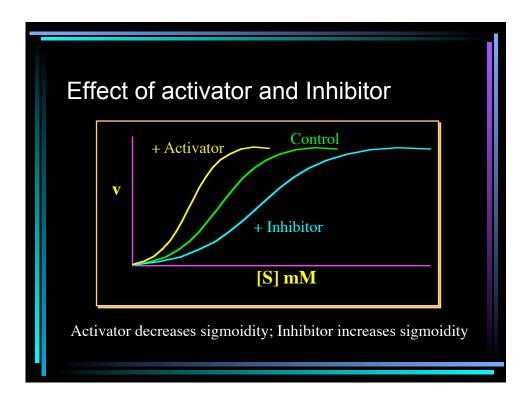
- Fructose-1,2-bisphosphatase
- NADP-Malate dehydrogenase
- Thylakoid ATP-synthase

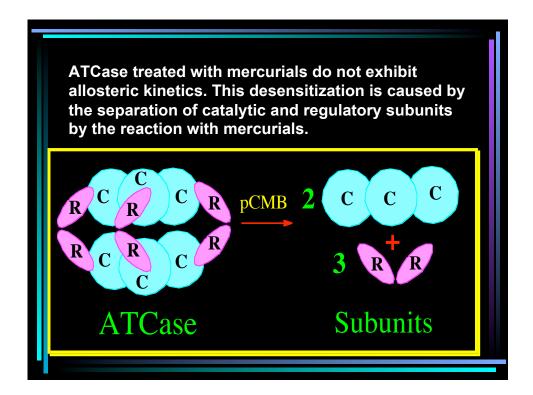
Inhibition by disulfide reduction

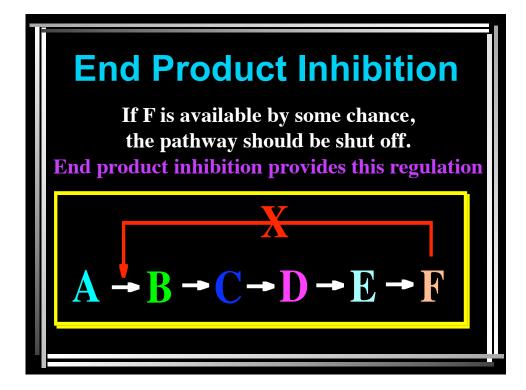
Phosphofructokinase

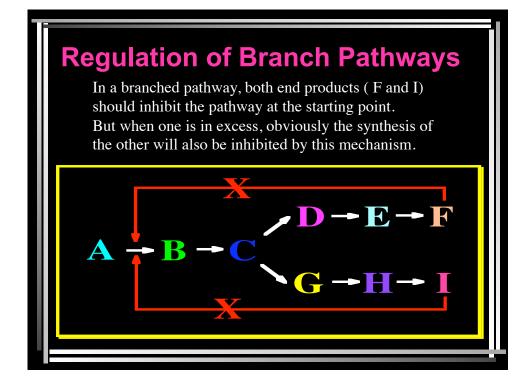


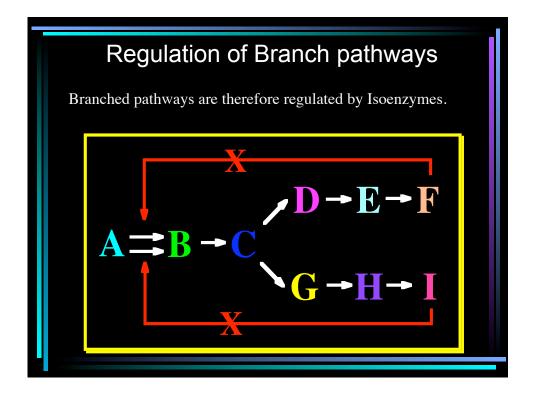


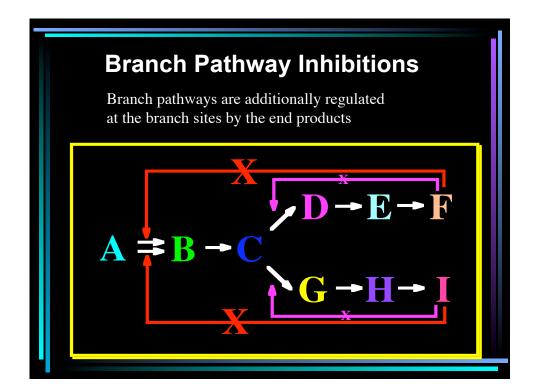


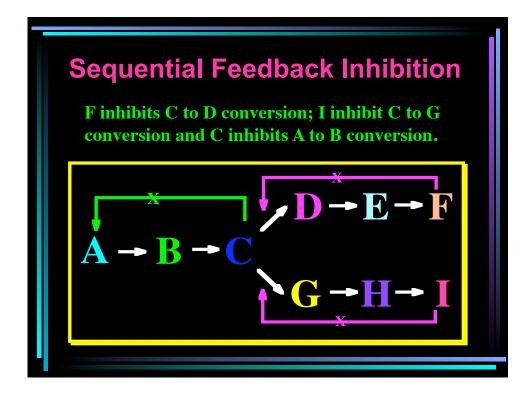


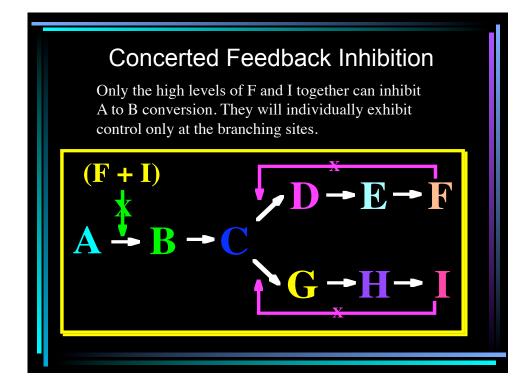


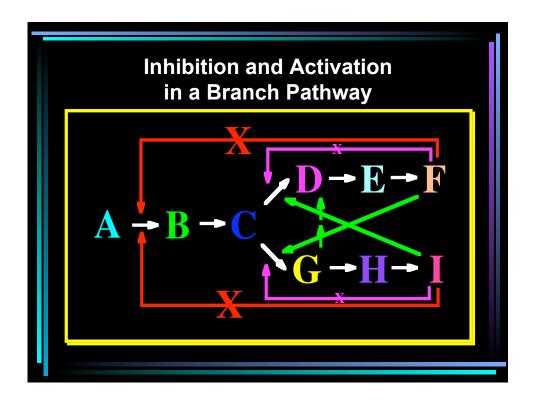












Cumulative Feedback Inhibition

The first enzyme is partly controlled by each end product.

Feed forward activation

$$A \xrightarrow{E1} B \xrightarrow{E2} C \xrightarrow{E3} D \xrightarrow{E4} E5 \xrightarrow{E5} F$$

$$\bigoplus$$

Excess B activates E4 so that the metabolic pathway goes forward.

Concerted model or symmetry model



The enzyme exists only in two states

The two states are T (taut or tensed) and R (relaxed)

Substrates and activators have great affinity for \boldsymbol{R} state

Inhibitors have higher affinity for T state

Ligands affect the equilibrium between T and R states

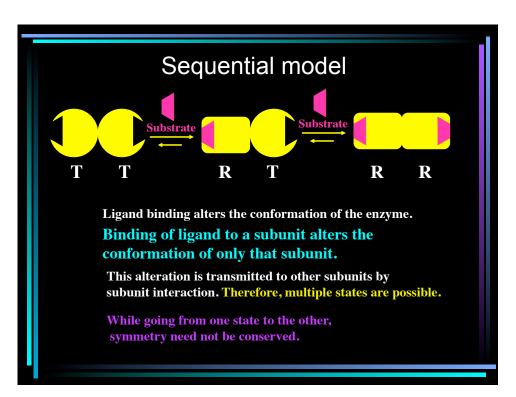
While going from one state to the other symmetry must be conserved.

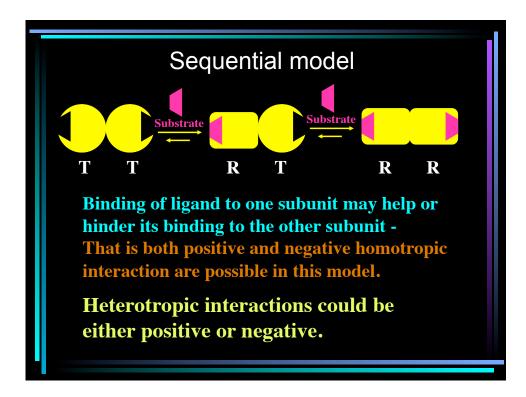
Concerted model or symmetry model

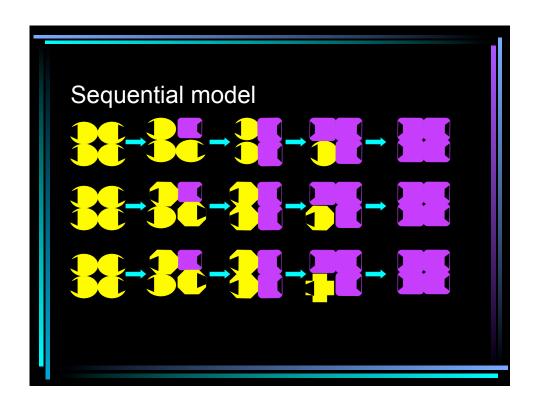


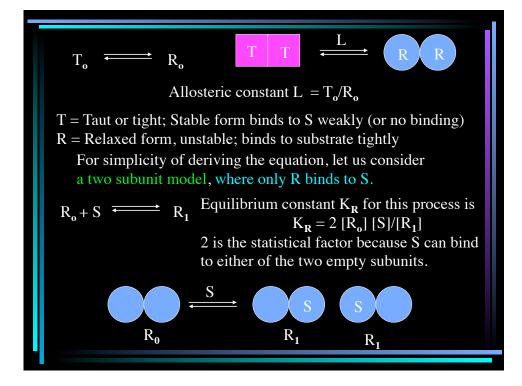
Binding of ligand to one subunit always assists the binding of the same ligand to the next subunit - This means that only positive cooperativity is possible.

Heterotropic interactions could be either positive or negative.









Allosteric models

Monod Wyman and Changeux model (MWC model)

Note:

K_R is the microscopic dissociation constant.

By reasons of symmetry, and because of the binding of Of any one ligand is assumed to be intrinsically Independent of the binding of any other, the $K_{\mathbf{R}}$ values Are the same for all homologous sites.

$$R_1 + S \rightleftharpoons R_2$$

Equilibrium constant for the second binding, $K_R = [R_1] [S]/2 [R_2]$ Statistical factor 2 appears in the denominator, as S can come out of either of the two bound subunits.

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Since, $K_{\mathbf{R}} = \overline{2} [R_{\mathbf{o}}] [S]/[R_{\mathbf{1}}]$ $[R_{\mathbf{1}}] = 2 [R_{\mathbf{o}}] [S]/[K_{\mathbf{R}}] = 2\alpha [R_{\mathbf{o}}]$ where $\alpha = [S]/[K_{\mathbf{R}}]$

Similarly, $[R_2] = [R_1]$ [S]/2 $[K_R]$ Or $[R_2] = [R_1]$ $\alpha / 2$ Substituting for $[R_1]$, we get, $[R_2] = \alpha^2$ $[R_0]$

Y = Fractional sites occupied by the ligand is defined as

$$Y = \frac{\text{Occupied sites}}{\text{Total sites}} = \frac{[R_1] + 2 [R_2]}{2 ([T_0] + [R_0] + [R_1] + [R_2])}$$

$$Y = \frac{2 \alpha [R_o] + 2 \alpha^2 [R_o]}{2 ([T_o] + [R_o] + 2 \alpha [R_o] + \alpha^2 [R_o])}$$

Dividing Nr and Dr by $2[R_0]$, we get

Y =
$$\frac{\alpha (1 + \alpha)}{([T_o]/[R_o]) + 1 + 2 \alpha + \alpha^2}$$

Since, L = [To] / [Ro] and $(1 + 2\alpha + \alpha^2)$ is $(1 + \alpha)^2$

The above equation simplifies to
$$Y = \frac{\alpha (1 + \alpha)}{L + (1 + \alpha)^2}$$

Y is related to velocity of an enzyme reaction. Since v is dependent on the fractional sites occupied by the substrate, $v = Y V_{max}$ or $Y = v/V_{max}$

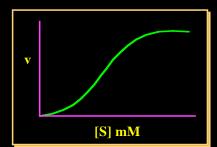
$$Y = \frac{\alpha (1 + \alpha)}{L + (1 + \alpha)^2}$$

This equation was derived with the assumption, that the enzyme has two subunits. If it has n subunits,

The equation is

$$Y = \frac{\alpha (1 + \alpha)^{n-1}}{L + (1 + \alpha)^n}$$

This equation defines a sigmoidal curve. A plot of Y (or v) versus α (or [S]) gives a sigmoidal graph.



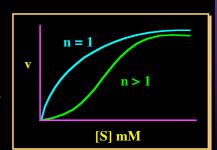
The sigmoidity depends on the values of n, and L.

$$Y = \frac{\alpha (1 + \alpha)^{n-1}}{L + (1 + \alpha)^n}$$

If n = 1 (there is only one subunit per enzyme molecule), the equation reduces to Michaelis - Menten type.

$$Y = \frac{\alpha (1 + \alpha)^0}{L + (1 + \alpha)^1} = \frac{\alpha}{L' + \alpha}$$

Hence, for sigmoidal curve, you need at least 2 subunits. Higher the value of n, higher will be sigmoidity.



Value of L $T_o \leftarrow R_o = L = T_o / R_o$

 Γ_{o}/R_{o} $Y = \frac{\alpha (1 + \alpha)^{n}}{\alpha}$

Let us assume $L = 10^4$,

For every one of R_o , there will be 10^4 of T_o

The addition of S causes removal of R_o from equilibrium So more and more T_o gets converted to R_o for S binding.

Since T--> R causes concerted changes in the structure of both subunits, the proportion of enzyme in R form increases progressively as more and more S are added. Thus binding of S is said to be Cooperative.

If an activator is present, say L is reduced to 10^3 (this means for every $10^3\,T_o$ there is one R_o), we need less S to shift the equilibrium to R state. Therefore, sigmoidity decreases if L is small. In the same way, if an inhibitor is present, say $L=10^5$, sigmoidity increases.

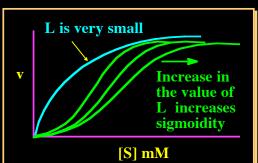
Therefore, higher the value of allosteric constant L, higher will be the sigmoidity and lower the value of L, lower will be sigmoidity.

If the value of L becomes very low, it can be ignored from denominator. So the equation again reduces to M. M. type:

In
$$Y = \frac{\alpha (1 + \alpha)^{n-1}}{L + (1 + \alpha)^n}$$

If L is omitted, the equation is

$$Y = \frac{\alpha (1 + \alpha)^{n-1}}{(1 + \alpha)^n} = \frac{\alpha}{1 + \alpha}$$



Effect of Activator and Inhibitor

If we add the effect of inhibitors and activators to this equation, We get:

$$Y = \frac{\alpha (1 + \alpha)^{n-1}}{L + (1 + \alpha)^n}$$

$$Y = \frac{\alpha (1 + \alpha)^{n-1}}{L \left\{ \frac{(1 + \beta)^n}{(1 + \tau)^n} \right\} + (1 + \alpha)^n} \quad \beta = K_I[I] \quad \tau = K_A[A]$$

$$\beta = K_{\mathbf{I}}[\mathbf{I}] \quad \tau = K_{\mathbf{A}}[\mathbf{A}]$$

Where

If $\beta = 0$, and $\tau = 0$ you get a normal sigmoidal graph. When an inhibitor is present, it stabilizes T state. β is increased and you get an increase in sigmoidity. When an activator is present, it stabilizes the R state, τ is increased and sigmoidity is decreased.

So far, we dealt with a special case where S binds to only R. If S also binds to T, but less effectively than it does to R, The equation for allosteric interaction becomes:

$$Y = \frac{\alpha (1 + \alpha)^{n-1} + LC \alpha (1 + C\alpha)^{n-1}}{(1 + \alpha)^{n} + L (1 + C\alpha)^{n}} \quad \text{Where } C = K_T / K_R$$

 K_T is the intrinsic binding constant for S to bind to T K_R is the intrinsic binding constant for S to bind to R

Lower the value of C (that is binding occurs only to R state As we originally considered) higher will be the sigmoidity. Higher the value of C, lower will be the sigmoidity.

Derivation of equation - Binding to both sites.

 $T_0 \leftarrow R_0$



 $\stackrel{L}{\longleftarrow}$



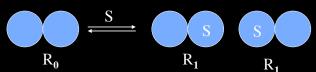
Allosteric constant $L = T_o/R_o$

T = Taut or tight; Stable form binds to S weakly

R = Relaxed form, unstable; binds to substrate tightly

 $R_o + S \longrightarrow R_1$ Equilibrium constant K_R for this process is $K_R = 2 [R_o] [S]/[R_1]$

2 is the statistical factor because S can bind to either of the two empty subunits.



$$R_1 + S \longrightarrow R_2$$

Equilibrium constant for the second binding, $K_R = [R_1] [S]/2 [R_2]$ Statistical factor 2 appears in the denominator, as S can come out of either of the two bound subunits.

Since,
$$K_R = 2 [R_o] [S]/[R_1]$$

 $[R_1] = 2 [R_o] [S]/K_R = 2\alpha [R_o]$ where $\alpha = [S]/K_R$

Similarly, $[R_2] = [R_1]$ [S]/2 $[K_R]$ Or $[R_2] = [R_1]$ $\alpha / 2$ Substituting for $[R_1]$, we get, $[R_2] = \alpha^2$ $[R_0]$

For T state:
$$T_o + S \leftarrow T_1 \qquad K_T = 2 [T_o] [S]/[T_1]$$

$$[T_1] = 2 [T_o] [S] / K_T$$

Let us assume $c = K_R/K_T$

Therefore, $K_T = K_R/c$, the above equation becomes,

$$[T_1] = 2 [T_0] c [S]/K_R$$

Since,
$$\alpha = [S]/K_R$$

We can reduce this further to $[T_1] = 2 [T_o] c \alpha$

On the same reasoning, $[T_2] = 1/2 \{ [T_1] [S] / K_T \}$

$$[T_2] = \frac{2 [T_o] c \alpha [S]}{2 K_T} = \frac{[T_o] c \alpha [S] c}{K_R} = [T_o] (c \alpha)^2$$

$$Y = \text{Fractional sites occupied by the ligand} \quad Y = \frac{\text{Occupied sites}}{\text{Total sites}}$$

$$Y = \frac{([R_1] + 2 \ [R_2]) + ([T_1] + 2 \ [T_2])}{2 \left\{([R_o] + [R_1] + [R_2]) + ([T_o] + [T_1] + [T_2])\right\}}$$

$$Substituting \ R_1 = 2\alpha \ [R_o]; \ R_2 = \alpha^2 \ [R_o]; \ T_1 = 2c\alpha \ [T_o]; \ and \ T_2 = (c\alpha)^2 \ [T_o];$$

$$Y = \frac{2\alpha \ [R_o] + 2\alpha^2 \ [R_o] + 2c\alpha \ [T_o]; + 2c\alpha \ [T_o]; + 2(c\alpha)^2 \ [T_o]}{2 \left\{([R_o] + 2\alpha \ [R_o] + \alpha^2 \ [R_o]) + ([T_o] + 2c\alpha \ [T_o] + (c\alpha)^2 \ [T_o])\right\}}$$

$$Y = \frac{2\alpha \ [R_o] \ (1+\alpha) + 2c\alpha \ [T_o] \ (1+c\alpha)}{2 \left\{[R_o] \ (1+\alpha)^2 + [T_o] \ (1+c\alpha)^2\right\}}$$
 Canceling 2 and dividing by $[R_o]$
$$Y = \frac{\alpha \ (1+\alpha) + c\alpha \ [T_o]/[R_o] \ (1+c\alpha)}{(1+\alpha)^2 + [T_o]/[R_o] \ (1+c\alpha)^2}$$
 Since $L = [T_o]/[R_o]$
$$Y = \frac{\alpha \ (1+\alpha) + L \ c\alpha \ (1+c\alpha)}{(1+\alpha)^2 + L \ (1+c\alpha)^2}$$

General equation

$$Y = \frac{\alpha (1 + \alpha) + L c\alpha (1 + c\alpha)}{(1 + \alpha)^2 + L (1 + c\alpha)^2}$$

This equation is for 2 subunit case. If we have n subunits, the equation can be modified as

$$Y = \frac{\alpha (1 + \alpha)^{n-1} + L c\alpha (1 + c\alpha)^{n-1}}{(1 + \alpha)^n + L (1 + c\alpha)^n}$$

If c = 1 that is if S binds to both T and R states equally well the equation reduces to

$$Y = \frac{\alpha (1 + \alpha)^{n-1} + L \alpha (1 + c\alpha)^{n-1}}{(1 + \alpha)^n + L (1 + c\alpha)^n} = \frac{(1 + L) \alpha (1 + \alpha)^{n-1}}{(1 + L) (1 + \alpha)^n}$$

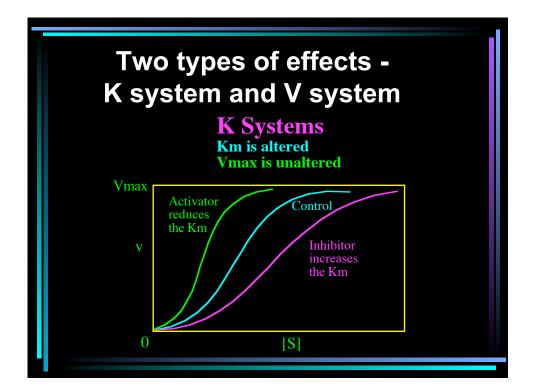
Y becomes = $\alpha/(1+\alpha)$ or Michaelis Menten equation

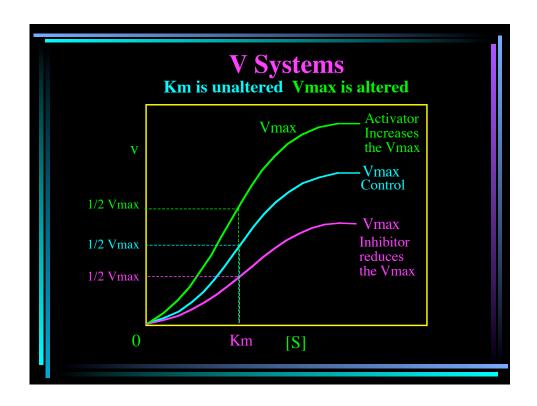
$$Y = \frac{\alpha (1 + \alpha)^{n-1} + L c\alpha (1 + c\alpha)^{n-1}}{(1 + \alpha)^n + L (1 + c\alpha)^n}$$

If L is approaching $\,0$ (that is the equilibrium is largely in favor of R state), we can omit the terms containing L as they will be very small and the equation reduces to Michaelis Menten equation again.

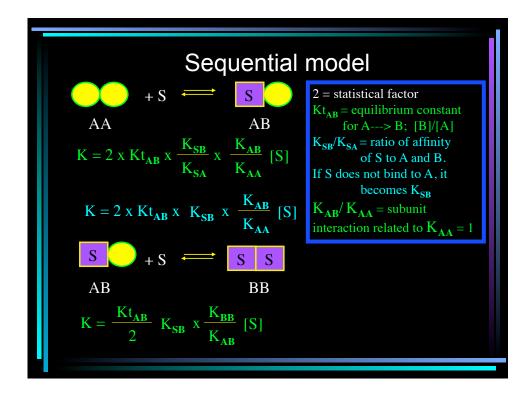
$$Y = \frac{\alpha (1 + \alpha)^{n-1}}{(1 + \alpha)^n} = \alpha/(1 + \alpha)$$

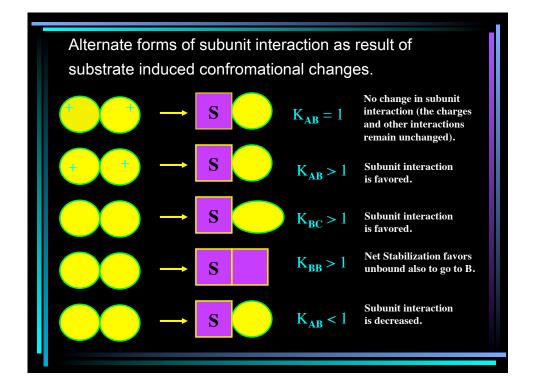
Thus larger the value of L, large will be sigmoidity Lower the value of L, lower will be the sigmoidity.





Sequential model - equation



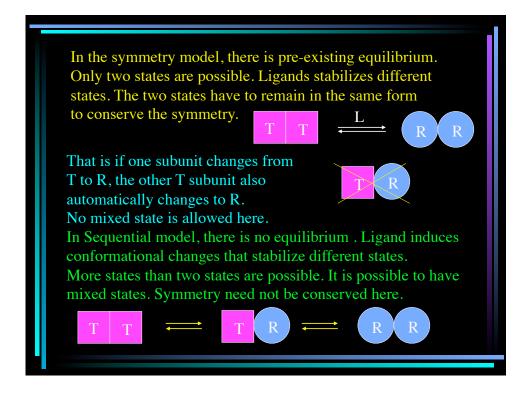


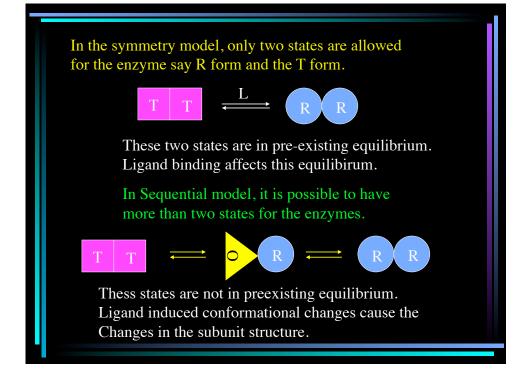
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If K_{BB}/K_{AB} is >> K_{AB}/K_{AA}, binding of S is increased to the second in comparison to the first Hence it is positive cooperativity. (There is net stabilization, K_{AB} = 1 and K_{BB} > 1)
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If K_{BB}/K_{AB} is $<< K_{AB}/K_{AA}$, binding of S is decreased to the second in comparison to the first Hence it is negative cooperativity. (BB shows repulsion $K_{BB} < K_{AB} = 1$)

If K_{BB}/K_{AB} is $= K_{AB}/K_{AA}$, binding of S is same to the second as that of the first Hence no cooperativity. $K_{AB} = K_{BB} = 1$) Michaelis Menten type kinetics

Differences between models





In the Symmetry model only positive homotropic interaction is allowed. [S binding always helps another S binding] Heterotropic interactions could be positive or negative. [S and I binding is negative heterotropic interaction and S and A binding is positive heterotropic interaction].

In the Sequential model, both homotropic and heterotropic interactions could be positive or negative.

[That is even S binding to one subunit could inhibit The S binding to the next subunit (negative homotropic interaction is possible here).

Difference between two models

Concerted model (MWC model)

Ligands stabilizes them Symmetry should be conserved Only two states are allowed. All transitions occur simultaneously Predicts M. M. kinetics if the subunits are in the active form Simple. Restricted application Same states could stabilized by different ligands (A & S \rightarrow R) Homotropic interaction are only positive

Sequential model (KNF model)

Based on Pre-existing equilibrium Based on induced fit theory. Ligand induces conformational changes No symmetry. Mixed states allowed Many states are possible. Different ligands stabilize differently M. M. kinetics only when there is no change in subunit interaction Complex. General application. Different states are stabilized by different ligands slightly. Homotropic interactions could be both positive and negative.

Hill's equation

Hg
$$(O_2)_4 \longrightarrow Hg + 4O_2$$
 ES₁₁ ES₁₂ ES₁₃ ES₁₄ ES₁₅ ES₁₆ ES₁

 $ES_n \Longrightarrow E + nS$

$$Y = \frac{[S]^n}{n}$$

$$1 - Y = 1 - \frac{[S]^n}{K^n + [S]^n}$$
 or $= \frac{K^n}{K^n + [S]^n}$ Equation 2

Divide equation 1 by 2
$$\frac{Y}{1-Y} = \frac{[S]^n}{K^n}$$
Equation 3

log of equation 3 yeilds the Hill's equation:

$$\log \frac{Y}{1 - Y} = n \log [S] - n \log K$$

Hill's plot

Hill's Plot

(Slope = n)

Log S

n = 1 - For Michaelis Menten type

n < 1 - Negative cooperativity

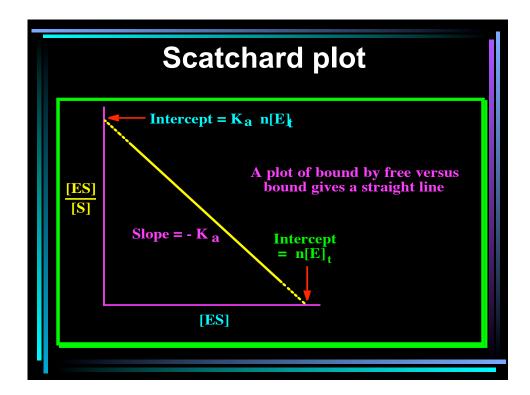
n > 1 - Positive cooperativity

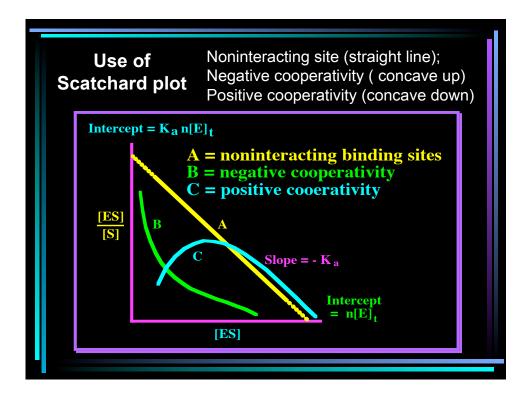
(Myoglobin n = 1; Hemoglobin n = 2.8)

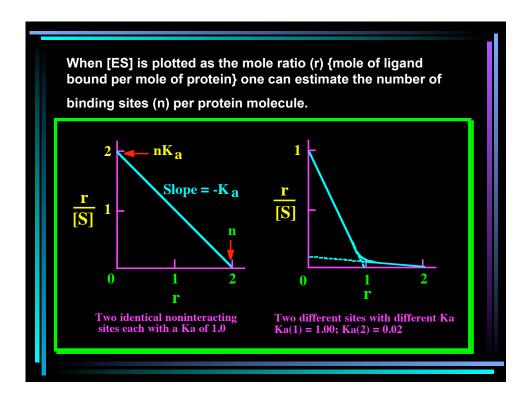
Scatchard plot

$$E + S = ES \quad Ka = \frac{[ES]}{[E]_o[S]} \text{ equation 1}$$
Concentration of bound total E is where n is total number of sites
$$= n[E]_t \quad \text{equation 2}$$
According to law of Mass action, $[E]_o = n[E_t] - [ES] \text{ equation 3}$ (unbound = total - bound)

Substitution equation 3 in 1, $Ka = \frac{[ES]}{(n[E]_t - [ES])[S]}$
or $Ka(n[E]_t - [ES]) = \frac{[ES]}{[S]}$
SCATCHARD PLOT:
$$\frac{[ES]}{[S]} = Kan[E]_t - Ka[ES]$$
A plot of bound by free versus bound gives a straight line.







$$R_{\mathbf{S}} = \frac{\text{Substrate concentration at 0.9 saturation}}{\text{Substrate concentration at 0.1 saturation}}$$

Since $Y = [S]/K_m + [S]$, the value at 90% and 10% saturation are

$$0.9 = [S_{0.9}] / K_m + [S_{0.9}] \quad \text{and} \quad 0.1 = [S_{0.1}] / K_m + [S_{0.1}]$$

Therefore,
$$0.9 \text{ K}_{\mathbf{m}} + 0.9 [S_{\mathbf{0.9}}] = [S_{\mathbf{0.9}}]$$
 or $0.9 \text{ K}_{\mathbf{m}} = 0.1 [S_{\mathbf{0.9}}]$ or $[S_{\mathbf{0.9}}] = 9 \text{ K}_{\mathbf{m}}$

$$0.9 \text{ K}_{\text{m}} = 0.1 [\text{S}_{0.9}] \text{ or } [\text{S}_{0.9}] = 9 \text{ K}_{\text{m}}$$

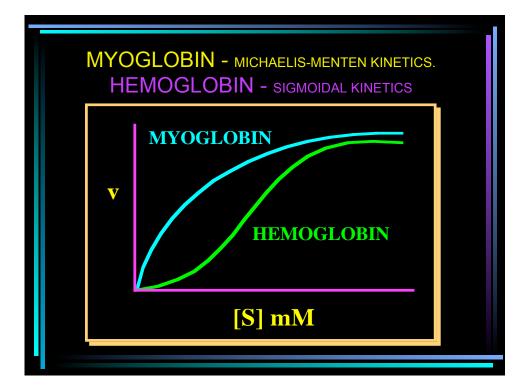
Similarly, 0.1
$$K_m + 0.1 [S_{0.1}] = [S_{0.1}]$$
 or $0.1 K_m = 0.9 [S_{0.1}]$ or $[S_{0.1}] = 1/9 K_m$

Hence,
$$R_s = \frac{9 K_m}{1/9 K_m} = 81$$
 for Michaelis Menten type.

 $R_s > 81$ for negative cooperativity

Rs < 81 for positive cooperativity.

Example of allosteric enzyme

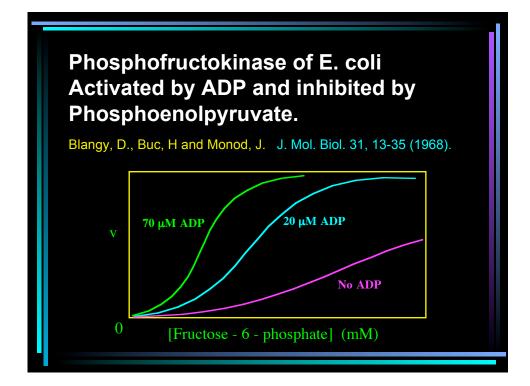


Myoglobin

- Monomeric in nature.
- M.M. Kinetics.
- pH has no drastic effect.
- CO₂ No effect.
- Diphosphoglycerate No effect
- Only one form.

Hemoglobin

- Tetrameric in nature
- Two kind of subunits. $\alpha_2\beta_2$.
- · Sigmoidal kinetics.
- pH inhibits O₂ binding.
- CO₂ inhibits O₂ binding.
- Diphosphoglycerate inhibits O₂ binding.
- Exists in two forms oxy form is different from deoxy form.



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