ABSTRACT

Absorbing boundary conditions for the finite-difference time evolution of the Wigner function using a constant flux condition through the 2D region of calculation in phase space are presented. Numerical results on the time evolution of a Gaussian wave packet in phase space will be presented and discussed. The conditions minimize undesirable reflections at the artificial boundaries of the computation.

BACKGROUND

Phase-space density holes are vortex-like nonlinear structures that have been observed in the magnetosphere. To study the time evolution of these structures we evolve the Wigner function in time by solving the Vlasov equation using finite-differences. Wigner function is a quasi-probability density function in phase space.

EQUATIONS

In one-dimensional case, the Wigner Function is:

\[ W(x,p, t) = \frac{1}{\pi \hbar} \int e^{i px - \frac{1}{2} \hbar \partial_x^2} \psi^* \psi \, dx \]

It satisfies the Vlasov equation:

\[ \frac{\partial W}{\partial t} + \frac{\partial W}{\partial x} \frac{\partial \psi^*}{\partial x} - \frac{\partial W}{\partial p} \frac{\partial \psi}{\partial p} = 0 \]

CALCULATIONS

Initial Conditions:

We start from a Gauss function which is symmetric both with position and momentum:

\[ \psi(x,t) = \frac{1}{\sqrt{\pi \hbar^2}} e^{-\frac{x^2}{2\hbar^2}} \]

Calculating:

By using finite-difference approximation, the Wigner function can be evolved step by step in time:

\[ W_{n+1} = W_n + \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

where \( \Delta x \) is the flux that went through the boundary (N+1)\( \Delta x \) in the flux that went through N\( \Delta x \) with a time shift T:

\[ \psi(x,t) = \frac{1}{\sqrt{\pi \hbar^2}} e^{-\frac{(x+\Delta x)^2}{2\hbar^2}} \]

Similarly we have:

\[ W_{n+1} = W_n + \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

where \( \Delta \psi \) is the group velocity.

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\[ W_{n+1} = W_n + \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

at v boundary, where \( \Delta \psi \) is the group acceleration.

If we assume the flux (\( \Delta x \)) that goes through the boundary \( \Delta \psi = \Delta x \) is the flux that went through N\( \Delta x \) with a time shift T:

\[ (\Delta x \Delta x) = \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

which means:

\[ \Delta x \Delta x = \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

where V is the group velocity.

Similarly we have:

\[ \Delta x \Delta x = \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

at v boundary, where \( \Delta \psi \) is the group acceleration.

In the case \( \Delta \psi = 0 \), we can assume:

\[ \Delta x \Delta x = \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

we can rewrite it in another form:

\[ (\Delta x \Delta x) = \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

which means:

\[ \Delta x \Delta x = \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

where \( \Delta \psi \) is the group velocity.

Similarly we have:

\[ \Delta x \Delta x = \Delta t \frac{\partial^2 W_n}{\partial x^2} \]

CONCLUSIONS

- It turns out we get the same absorbing boundary conditions using a dispersion relation for plane waves or using a constant flux assumption.

- The remaining small reflection is due to the first-order finite difference approximation, so we can reduce it by using higher resolution on x, v or t.

REFERENCES

