Fostering Mathematical Habits of Mind:

How Parents Can Support

Their Children’s Mathematical Thinking in the Home

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An Orientation to the Issue: Mathematics Education Reform

Improving mathematics teaching and learning is a top priority on our nation’s education agenda. A strong foundation in mathematics is essential for knowledge-based technological economy of the 21st century. Studies have shown that mathematics is the strongest predictor of college completion (Adelman, 1999) and, critical for college and workforce readiness is student success in algebra – a prerequisite “gatekeeper” to higher-level mathematics courses (Achieve, 2008; ACT, 2006; Adelman, 2006).

Over the past 25 years, our nation has strived to raise the mathematical proficiency of all students. “A Nation at Risk” report in 1983 warned of rising levels of mediocrity in American schools and global competitiveness, adding up to an educational crisis in America. From this national imperative, systemic reform era was born. The National Council of Teachers of Mathematics (NCTM) developed new common standards and assessments set expectations for what students should learn in pre-college mathematics (1989, 1991, 2001). The federal No Child Left Behind Act (2001) mandated public school accountability measures and annual progress goals. And, the National Mathematics Advisory Panel Report (2008) called for an increased focus on algebra in K-8. These comprehensive policy efforts were designed to affect mathematics teaching practices and training, curriculum content, and the capacity of the education community to improve mathematics proficiency for all students.

Results from the nation’s report card, or National Assessment of Educational Progress (NAEP) indicate the mathematics reform is making a difference. Since 1990, fourth-graders who scored at or above proficient in mathematics has tripled, from 13 percent to 39 percent (Chandler, 2007, Sept. 26; NCES, 2007); however, these gains are not widespread. Achievement gaps persist between minority and low-income groups and their peers, and performance is not increasing for twelfth graders. A fall off in mathematics achievement is evident as students enter middle and high school, when the mathematics becomes more difficult. NAEP data shows that at least one-third of the nation’s eighth and twelfth grade students have below-basic mathematics skills (Lee, Grigg, & Dion, 2007; US Department of Education, 2005). Furthermore, from this same data, students across grades 4, 8, and 12 who agreed they enjoyed math and found it useful outperformed those who disagreed with these attitudes (NCES, 2000).
These findings indicate that students are not graduating from high school with mathematical skills and content knowledge they need for the demands of the 21st century workforce (ACT, 2006; Achieve and National Governors Association, 2005; NMAP, 2008). Their mathematical unpreparedness is a critical concern for K-16 educators, employers, and parents. Beyond the policies and practices of our education system, what can the education community do to meet the challenges of improving children’s mathematics proficiency?

**Fostering Mathematical Habits of Mind in the Home**

This report will address this question by focusing on a particular out-of-school context in which all children have access to: the home. In particular I emphasize parent involvement in mathematics and the positive behaviors and skills parents can foster in the home to help students succeed in mathematics.

I propose a *mathematical habits of mind* framework for parent involvement as a means to support children in mathematics learning. There are research-based practices for parent involvement in mathematics in which parents, teachers, and administrators can benefit. As this is a work-in-progress, I intend to build this model and with the mathematics educators in my work at Education Development Center EDC, develop it into a grant proposal to provide workshop and educational resources for school districts to integrate into their parent outreach in mathematics.

**What are Habits of Mind?**

Habits of mind in education can be traced to the progressive educational philosophy of John Dewey, who emphasized the role of learners’ experiences, attitudes, and interests in the subject matter. In his view, the product of education is the formation of habits, and rote learning is its byproduct. Habits are ways of knowing. Dewey thought of habits as active not fixed or always negative; they are a way of gaining control of the environment through one’s control of thought and action. “A habit is a form of executive skill, of efficiency in doing. A habit means an ability to use natural conditions as means to ends (p. 30) … It means formation of intellectual and emotional disposition as well as an increase in ease, economy, and efficiency of action. (p. 31)” (Dewey, 1922, ch. 4). Habits
are the tendencies one has to think or act in a certain way, such as when finding one’s way in a foreign country.

A number of researchers’ definition of HoM reflects Dewey’s: as mental habits to develop self-regulated thinking and learning (Marzano, 1992); as a transcendent, multi-disciplinary set of intellectual virtues that incline one to think critically (Facione & Facione, 1994); and a pattern of intellectual behavior that leads to productive actions (Costa & Kallick, 2000). Perkins, Jay, and Tishman of Project Zero at Harvard University proposed that dispositions is the interplay of inclination, perceptual sensitivities, and abilities, and defined seven dispositions necessary for good thinking (1993).

**Why is Thinking Skills Necessary in Mathematics?**

Mathematics is important not only because it’s a core subject area in K-12 education and a “gateway” to college and career readiness and to higher-order thinking skills (Achieve, 2008). Mathematics is the foundation for abstract, applied skills such as critical thinking, complex reasoning, research and analytical skills, problem solving, and communication (e.g., American Diploma Project, 2004; Achieve, 2008). The notion that advanced analytical skills or “21st century skills” should be integrated into K-12 teaching and learning in addition to the basic skills and core content areas—and that these higher order skills can be best learned together with the fundamentals, even in the early grades—is gaining prominence among the nation’s educational policymakers (e.g., Partnership for 21st Century Skills, 2008). The belief that advanced thinking skills, i.e., knowing how to apply and use content knowledge is equally if not more important than the knowledge itself is at the heart of the mathematics reform efforts and preeminent progressive thinkers such as Jerome Bruner.

> *We teach a subject not to produce little living libraries on that subject, but rather to get a student to think mathematically for himself, to consider matters as an historian does, to take part in the process of knowledge-getting. Knowing is a process, not a product.)*

(Bruner, 1966, p. 72)
Thus the significant shift in mathematics education is the inter-related mathematics content and mathematical thinking skills, moving away from the traditional skills to a more process-oriented constructivist approach to figuring things out.

**What are Mathematical Habits of Mind?**

Constructivist perspectives posit that learners actively construct mathematical knowledge in much the same way mathematicians use the problem solving process to create new knowledge (Baroody, 1987). The habits of mind used by mathematicians, such as making conjectures, experimenting, and looking for patterns (Cuoco, Goldenberg, & Mark, 1996), is the basis for a high school mathematics curriculum funded by the NSF and developed at EDC (2009). The textbooks are organized around the habits of mind that underlie high school mathematics:

*The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking—the habits of mind—used by scientists, mathematicians, and engineers, and used in other professions in which mathematics is a core ingredient.* (Cuoco, 2006).

This emphasis on the scientific process as opposed to the mastery of mathematical topics is prominent in inquiry-based science and engineering education as well. Habits of thought are recognized in the NCTM Principles and Standards and American Academy for the Advancement of Science (AAAS) Benchmarks.

The NCTM Standards (1989) describe the overarching goal of developing students mathematical power, or “the ability to use information to reason and think creatively and to formulate, solve, and reflect critically on problems.” To assess students’ mathematical knowledge the NCTM recommends evaluating their mathematical dispositions when approaching mathematical tasks and evidence that teachers foster them (2001). The AAAS Benchmarks assert that students should learn to evaluate scientific claims and to explicitly reflect on the problem solving experience in order to generalize these skills to new contexts (1993, p. 282). For example, an AAAS benchmark for grades 6-8 is that students should present a scientific idea using reasoning skills and evidence to support their claims.

As teachers implement new approaches to develop students’ mathematical thinking, approaches that are substantially different than what many adults experienced in their
mathematics education, they are re-conceptualizing their beliefs about mathematics and pedagogy, acting as both change agents and products of reform (Anderson & Piazza, 1996). Professional development programs are immersing teachers in mathematics, allowing them to experience the discipline as a learner and reflect on the experience as a teacher.

**Mathematics and Culture**

Learning to think mathematically requires that students and teachers develop a mathematical perspective, a mathematical way of seeing the world (Schoenfeld, 1992). From a sociocultural constructivist perspective, mathematics learning is shaped not only by instruction but also by social interactions and activities within one’s culture. One hypothesis is that learners’ develop mathematical habits, skills, and meaning through a process of enculturation (Resnick, 1989 in Schoenfeld, 1992). Building on Perkins et al. (1993) conception of habits of mind, a culture can demonstrate that it values mathematical thinking by creating contexts for students to develop and use these skills, and more importantly, help them develop the sensitivities to apply them appropriately and the inclinations to do so (Grotzer, 1996).

Cognitive research has shown students’ informal learning experiences have a strong effect on learning and problem solving skills (Cai, 2003). Students’ informal strengths and intuitive knowledge can dually function as a bridge for assimilating abstract school mathematics and a source for affective outcomes such as self-efficacy (Baroody, 1987, p. 35). Teachers can build on students’ everyday practical knowledge of mathematics to make mathematics more interesting and meaningful for the learner (Hiebert, 1984 in Baroody, 1987).

**Mathematical Habits of Mind and Parent involvement in Education**

At the heart of children’s socialization are parents. Parents are our children’s first teachers. A significant number of research studies have shown convincingly that children at all grade levels are more successful in school when their parents participate at school and encourage education and learning at home (Cotton & Wikeland, 2001; Grodnick & Slowiaczek 1994). Studies have linked parental involvement with student benefits of higher grades and test scores,
enrollment in more challenging academic programs, increases in the number of classes passed and credits earned, better attendance (Henderson & Mapp, 2002), and reduced student drop out rates (Rich, 1985). Studies also show parent involvement is related to affective outcomes of improvements in students’ attitudes, social behavior (Cotton & Wikelund, 2001), and motivation to learn (Epstein, 1992). These diverse outcomes include, but are not limited to, mathematics (Sheldon & Epstein, 2005).

“Parent involvement” encompasses a broad range of behaviors and activities as defined by research literature. One frequently cited framework to organize these activities is Epstein’s typology of parent involvement, conceived as a theory of overlapping “spheres of influence” or interrelationships of family, community, school, and peer groups. These spheres affect the nature and characteristics of parent involvement and its effect on students. Epstein’s six types are parenting, communicating, supporting school, learning at home, decision-making, and collaborating with community (adapted from Catsambis, 1998).

Which types more frequently lead to improved student outcomes? Research building on this typology, including a research synthesis of parent involvement practices (Henderson & Mapp, 2002), found programs that engage families in supporting their children’s learning in the home are linked to higher student achievement (Sheldon & Epstein, 2005; Catsambis, 1998), with activities that are explicitly linked to mathematics learning in the home having a greater impact than more general forms of parent involvement (Henderson & Mapp, 2002; Sheldon & Epstein, 2005). Such activities may include students’ talking about mathematics as part of their homework assignment or discussions about the use of mathematics in daily life. Additionally, Sui-Chu and Willms conducted a large-scale study of 1000 schools (1996), focusing on four patterns of parent involvement: home discussion, home supervision, school participation, and school communication, found that home discussion (e.g. talking about the school day, planning education programs) was the most significant factor in children’s academic achievement, after family background. A meta-analysis of parent involvement studies (Fan & Chen, 1999) found that parents’ high aspirations consistently had the greatest effect on student achievement at both primary and secondary levels. This is especially true for adolescents. Catsambis (2001) studied parent involvement activities of secondary students (using large-scale data) and found that high levels of parents’ educational expectations had the strongest effect on seniors’ achievement, regardless of social or ethnic background.
More closely related to fostering students’ mathematical habits of mind, a comparative analysis of US and Chinese parent involvement practices and sixth-grade student performance on non-routine mathematical problem solving revealed parent roles as *motivators* (emotional support), and *monitors* (response to teacher requests) were the strongest predictors of US students’ problem solving performance. These *indirect roles* were more effective than the direct role of content advising (help on homework) in the home setting (Cai, 2003), indicating that parents’ helping their children with homework isn’t nearly as powerful as thought. Interestingly, a large number of parents surveyed said they do not know how to motivate their children, a need for which the author hopes will be addressed in future studies.

Another parent factor that influences student achievement is parent’s attitudes, beliefs and expectations. Studies have demonstrated that children’s self-efficacy and academic achievement is shaped by parents’ attitudes and values on the importance of mathematics. Parents’ perceptions of their children’s abilities (in mathematics and science), and their children’s future success, and gender stereotypes can impact children’s later self-perceptions, achievement values, and performance in these domains. (Jacobs & Bleeker, 2004; Jacobs & Eccles, 1992; Pedersen, Elmore, & Bleyer, 1986). In a sample of students from grades 5 – 11 and their parents, researchers found that parent’s beliefs were the key mediators and more directly related to children’s self-concept and achievement; in mathematics, these beliefs are more critical mediators than children’s own past performance in mathematics (Jacobs & Eccles, 1992).

Perhaps the most significant research from this line of study is Carol Dweck’s research on student motivation. Parent and student perception of what it means to be intelligent and students’ view of intelligence can affect their performance in mathematics (Dweck, 1989; Dweck & Legget, 1988). These studies demonstrated that students who think that intelligence is a fixed entity are more likely to be performance oriented rather than learning oriented; they want to look good rather than risk making mistakes while learning. Such students are prone to give up as the task becomes more difficult. In contrast, students who think that intelligence is malleable are more willing to struggle with challenging tasks and are more comfortable with risk. A 2007 study co-authored by Dweck supported these findings: seventh grade students who received instruction on the changing nature of intelligence were more likely to improve their mathematics performance while those who did learn about the nature of intelligence did not make gains (Chandler, 2009).
For schools and policymakers, parent involvement is both an “enabling and enhancing” variable (Hoover-Dempsey & Sandler, 1995). Parents are a resource who can reinforce mathematical thinking skills, perspectives, and values that teachers model in the classroom. However, some say the mathematics community has not established a mechanism for parents to engage in the reform. Because most parents learned mathematics in a more procedural “absorption” methods, they may not know how to access their children’s mathematics learning, or their mathematics support at home may be incongruent (and counteractive) to the constructivist approaches (Peressini, 1998). Karen Mapp of Harvard University has called for a shift from school-centric parent involvement to parent-centric approach in which practitioners open a discussion with parents about their needs and wants and figure out how to support the existing visible and invisible ways parents support their children’s education. Social factors can prevent parents from becoming involved, such as parents’ full-time jobs or cultural and language barriers. Mapp encourages schools to develop two-way partnership with parents, rather than inviting parents into schools, which presents obstacles for many families.

With increasingly diverse student populations and home cultures, and outside pressures of NCLB and state tests, the time could not be more critical to uncover research-based strategies to engage parents in improving students’ mathematics achievement. Psychology professor Laurence Steinberg (with Brown & Dornbusch, 1997) contends that America’s reform efforts have disproportionately focused on structural elements of the education system with little attention to out-of-school factors, arguing:

*We must transform the national debate over the causes and cures of our achievement problems from one about reforming schools to one about changing students’ and parents’ attitudes and behaviors.* (p. 15)

Given this extreme view and the extensive changes schools and teachers are undertaking to improve students’ “mathematical power”, what roles and behaviors are envisioned for parents?
Integrating a Habits of Mind Approach to Parent Involvement in Mathematics

Just as teachers create classroom environments that value thinking and create opportunities for students to use thinking skills to learn deeply, so too can parents create climates that value mathematics and allow students to practice thinking skills at home. This “culture of good thinking” encourages student inclinations to use thinking skills and develop positive dispositions towards mathematics. Students need these thinking tools to ably and effectively activate, apply, and deepen their knowledge in different contexts and to acquire and integrate new knowledge. Research has shown students will not learn thinking skills without explicit attention to helping them so (Grotzer, 1996). In order to internalize, generalize these habits or behaviors and apply them with automaticity, students need repeated practice and reinforcement of HoM across K-12 education (Costa & Kallick, 2000).

The following initial recommendations ideas for parents, based on research, for integrating and valuing mathematical thinking skills in the home culture.

1. Mathematics Ability is not Fixed: Parents can help students understand that mathematics is not something you are born with. Too often adults say, “I was never good at math” or “I was never any good at math” when encountering mathematical tasks in the home. These attitudes influence children at a formative age. The fact is that most people have the ability to learn math but success depends primarily on one’s belief in their ability and their efforts.

2. Goal Setting: Parents influence the goals that children adopt for themselves. By honoring and rewarding effort over performance, it is more likely students will persist and focus on task mastery versus a performance goal, contributing to a stronger self-efficacy. Parents can set high expectations and provide high levels of encouragement. Goals can be powerful motivators for students and intrinsically rewarding.

3. Persistence in Digging Deeper: A math teacher who I work with shared her experiences teaching a mathematics enrichment elective course for 8th grade students. In each term, students conduct mathematical research, using mathematical habits of mind to investigate
a question about a math concept or topic of interest. Early on in the term, students are quick to say, “I’m done”. They begin working at the superficial level and do have the self-monitoring to continue digging deeper into the mathematics. They begin at a superficial level, presenting mathematics by working out one solution and looking up information on the Internet. It takes a lot of instruction time, but by the end of the term, students can work independently and have begun to develop the habits as formulating questions, seeking alternative approaches, or looking for patterns in their data. This is an example of Polya’s problem solving principles (1973), that students can structure their thinking and develop problem solving skills by reassessing the solution and looking back at the pathway that led them to the answer. Parents can engage their children in this same metacognitive awareness in the home. Take a problem missed on homework, a problem of the week that may be too challenging, and help students identify alternative strategies for solving it. Or simply ask students to explain what they learned in mathematics that day. Communicating a complex concept helps students to refine their thinking. Help students articulate their mathematics ideas. Explaining abstract concepts is a difficult task.

4. Value Math: Parents can identify everyday opportunities to engage their children in mathematics learning, creating meaningful and interesting contexts for young children to explore and make decisions in mathematics (Jacobs & Bleeker, 2004). Research shows early learning experiences in the first six years of life have positive influence on children’s school success. Early learning experiences in mathematics could be created through routine cultural activities such as grocery shopping, planting a garden, or balancing a checkbook. Often parents do not perceive these routines as opportunities to provide children with access to and engagement in mathematics used by the community. It is important for parents to build mathematics into everyday activities that children already enjoy. Just as teachers create contexts to scaffold children’s thinking in the classroom, parents can structure learning tasks in the home with increasing demand.

5. Mathematical viewpoint: See the world from a mathematical point of view. Observe geometrical figures in the environment. Model the behaviors of finding patterns or using
evidence and asking questions to defend knowledge claims. Have fun playing with numbers or math games. Show there is a systematic way of approaching a challenging problem. Try a different strategy when something doesn’t work. Making these habits of mind explicit for children can open their eyes to the creativity and challenge inherent in mathematics.

**Future Plans and Hopes**

This habits of mind approach to parent involvement in mathematics builds on what parents are already doing in the home and empower all parents to help their children realize mathematical potential. In addition to a parent involvement program to communicate research-based strategies for fostering mathematical thinking, I will use this research to develop a workshop model that gives parents a first-hand experience of learning mathematics in an inquiry process. As more schools hold math nights each fall, and teachers re-learn mathematics in professional development, it is important to give parents the opportunity to reflect on their personal experiences and re-construct their role in support their children’s mathematics. This framework can provide a schema for teachers to organize their parent outreach practice around a shared vision for mathematics learning, drawing collaboration with parents in fostering habits of mind.
Bibliography


