A Statistical – Topographic Model for Mapping Climatological Precipitation over Mountainous Terrain

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Objectives:

- Introduction
- Methods for estimating areal precipitation
- The PRISM conceptual framework
- Description of the model
- Model evaluation
- Summary and future work

Introduction

Purpose: to develop a method for distributing point measurements of monthly and annual average precipitation to regularly spaced grid cells at regional and continental scales.

Result: PRISM (Precipitation-elevation Regression on Independent Slope Model)

- is well suited to regions with mountains terrain because it incorporates a conceptual framework that addresses the spatial scale and pattern of orographic precipitation

Paper: describes the PRISM modeling system and compares its advantages and limitation to kriging, elevationally detrended kriging, and elevational cokriging;

Methods for estimating areal precipitation

- Graphical:

  mapping of precipitation data

  Thiessen polygon estimation

- Topographical: correlation of point precipitation data with an array of topographic and synoptic parameters: slope, exposure, elevation, location of barriers, wind speed and direction;
- Numerical = the most communally used precipitation distribution method:

  interpolation: Inverse-distance weighting

The PRISM conceptual framework

PRISM incorporates a conceptual framework that addresses orographic scale and pattern across the landscape.

Main aspects:

a) Precipitation – elevation relationships
b) The spatial scale of orographic effects
c) The spatial patterns of orographic regimes

Precipitation –elevation relationship

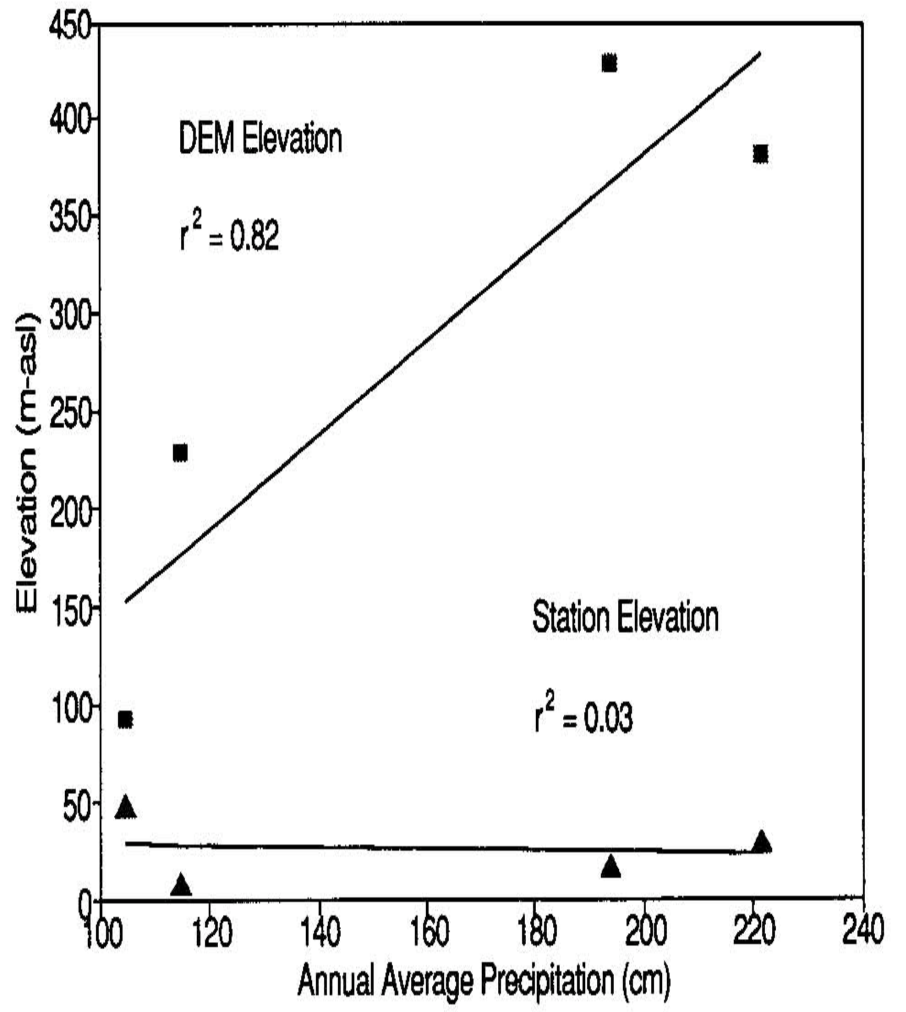
- PRISM assumes that precipitation on a given slope increases with elevation to a maximum at the mountain crest, unless local station data dictate otherwise.

The spatial scale of orographic effects

- PRISM calculates elevation of the stations from their position on the DEM:
  - How does the DEM grid resolution compare to the scale of orographic effects are tried to be simulated?
  - Is there an "optimal" resolution for orographic effect?

DEM gives excellent estimates on orographic effects

5-min latitude-longitude (6km x 9 km) DEM in many regions gives a better approximation of a station's orographic elevation than the station's actual elevation



The spatial patterns of orographic regimes

- It is essential that the topographic regions o be recognized and isolated;
- A mountain landscape can be divided into a mosaic of topographic faces, “facets”, each assumed to experience a different orographic regime;
- Each topographic facet is a contiguous area over which the slope orientation is reasonably constant;
- Topographic facets are best delineated by using a DEM with a resolution that closely matches the smallest orographic scale supported by data.

Description of the model

a) Overview

b) PRISM structure and function

c) FACET

d) PRISM

Description of the model
1. Overview

PRISM:

a) Estimates the "orographic" elevation of each precipitation station using a DEM at 5-min latitude-longitude grid spacing

b) Assigns each DEM grid cell to a topographic facet by assigning slope orientation.

PRISS estimation precipitation at each DEM cell by:

a) Using a windowing technique to develop a precipitation –DEM elevation regression function from nearby rainfall stations on the cell's facet;

b) Prediction precipitation at each cell's DEM elevation with this regression function

Description of the model

2. Structure and Function

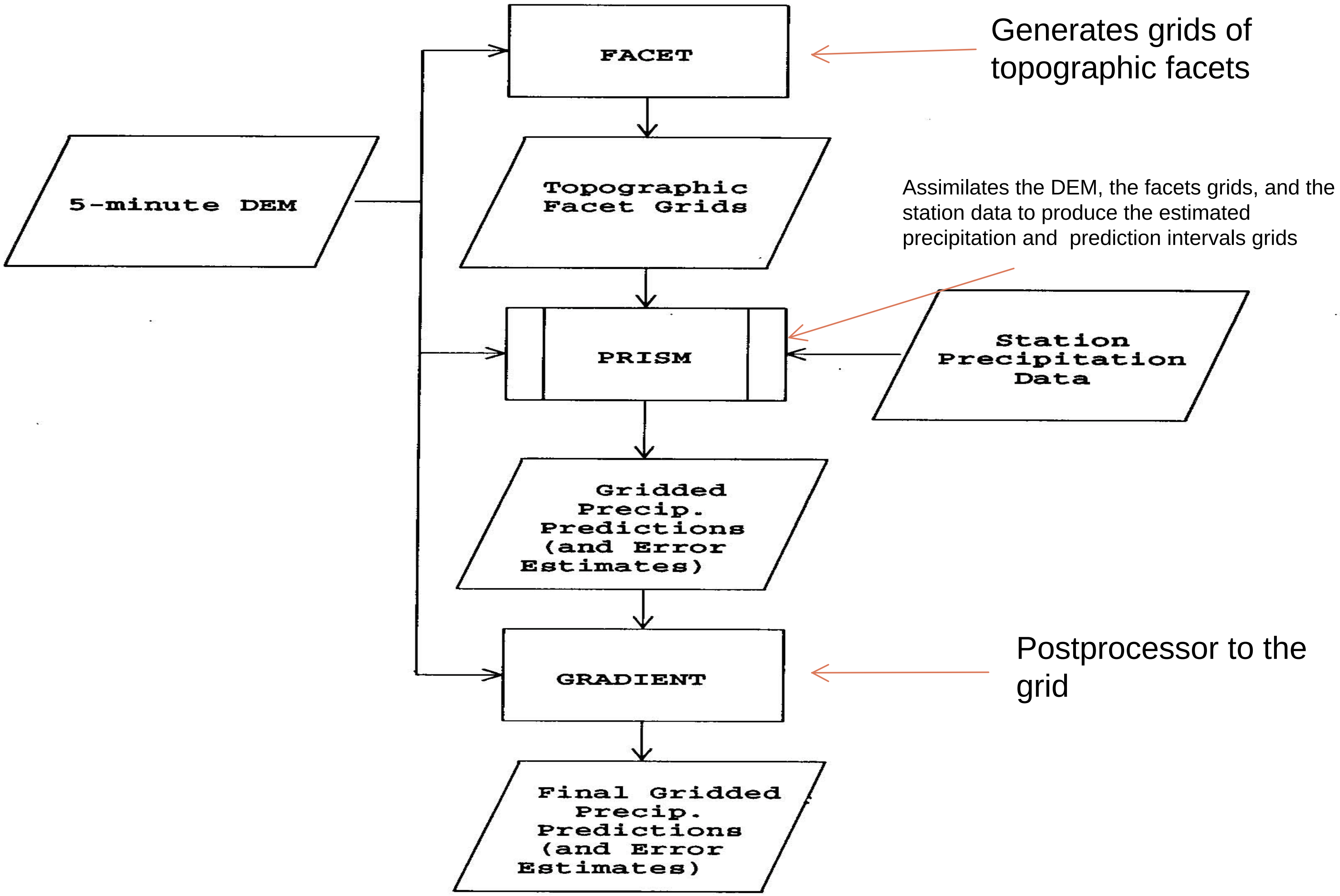


FIG. 2. Overall structure of the PRISM modeling system.

TABLE 1. Input parameters required by the PRISM system. Range, default, and comments on estimation method assume model is being applied to a DEM with 5-min latitude–longitude resolution.

Parameter	Definition	Units	Modeling system component	Range and default value	Method for estimation
MINSTP	Elevation gradient below which terrain is assumed to be flat	meters per grid cell	FACET	1–25 15	15 is used for most applications
MAXRAD	Maximum radius within which station search will be conducted	grid cells	PRISM	2–10 none	Cross validation (PSTAT)
MINRAD	Minimum radius within which all stations are included	grid cells	PRISM	2–5 none	Cross validation (PSTAT)
MINSTA	Minimum number of stations required for cell P/E* regression calculation	stations	PRISM	3–10 none	Cross validation (PSTAT)
B1MIN	Minimum allowable slope for cell P/E regression function	per kilometer	PRISM	0.0	0.0 is used for most applications
B1MAX	Maximum allowable slope for cell P/E regression function	per kilometer	PRISM	2.0–3.0 3.0	Largest P/E slope expected on a single facet in the domain
DB1	Default slope for cell P/E regression function	per kilometer	PRISM	0.6–1.3 none	Mean slope from all valid regressions in domain
B1EX	Extrapolation slope for cell P/E regression function	per kilometer	PRISM	1.5–2.5 2.0	Hold back high-elevation data and find B1EX giving best prediction stats
B1MAXG	Maximum allowable slope for a P/E relationship between cells	per kilometer	GRADIENT	2.5–5.0 4.0	The largest P/E slope expected between facets in the domain
MINSLP	Minimum elevation gradient needed to invoke GRADIENT operation	Meters per grid cell	GRADIENT	75–150 100	100 appears to be good for United States

* Precipitation–DEM elevation.

2.FACET:

Assigns a slope orientation to each DEM cell at column i, row j by first calculating elevation gradients across the cell from west to east and south to north

- The south –north gradient is calculated as the difference between adjacent cell (i, j-1) to the south and adjacent cell (i, j+1) to the north

The slope is flat if neither the west-to-east nor south-to-north gradients are equal or greater than a prescribed gradient (MINSTP)

If the slope is not flat, the cell is categorized as facing north, south, east or west

Description of the model

FACET ...

Data points on a facet can be extent by subjecting the DEM to a standard five-point filter in which the cell elevation is:

$$E_{ij} = 0.5E_{ij} + 0.125(E_{i+1\,j} + $$

FACET computes six different facet grids:

1)Derived from the unfiltered DEM
2)The DEM is subject of 8 filtering passes
3)16 passes
4)24passes
5)32 passes
6)40 passes

Facets that are only one cell in width are assimilated into the surrounding facets because at least 2 grid cells are needed.

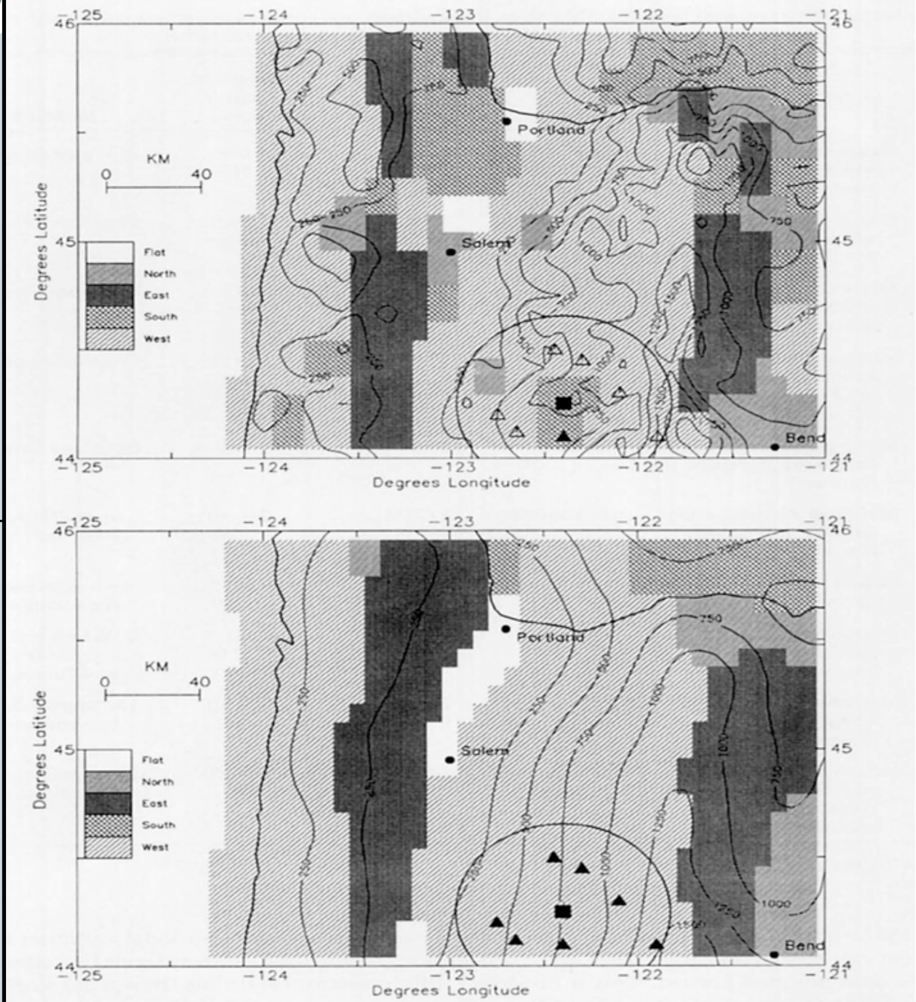


FIG. 3. Northwestern Oregon topographic facets produced by FACET and an example search window used by PRISM for (a) the unfiltered DEM and (b) the DEM filtered 40 times. Solid square denotes an example grid cell for which precipitation is to be estimated by PRISM. Open triangles are stations that fall within the grid cell's window but are rejected because they lie on a different topographic facet. Solid triangles are stations accepted for developing the precipitation–DEM elevation regression function for the grid cell.

Description of the model

3. PRISM

‘Orographic elevations for each
5-min DEM by:

- Locating the DEM grid cell cen
location;
- Finding the four surrounding g
- Calculating a weighted mean e
cell weighed inversely by its ce

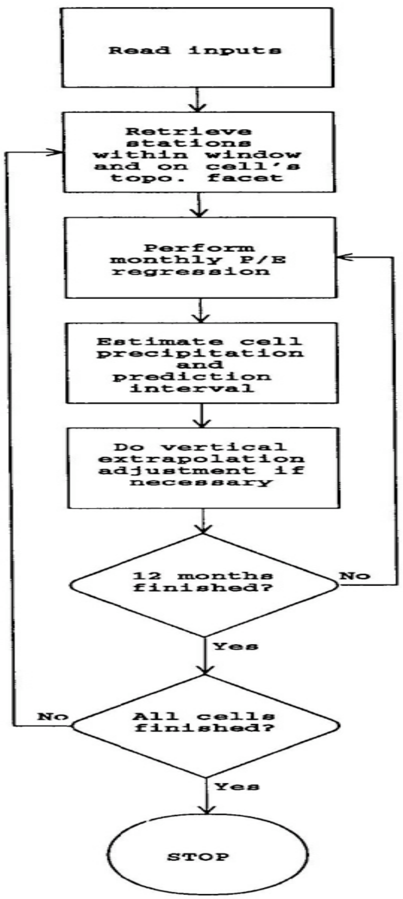


FIG. 4. Process flow for monthly operation of the main PRISM model.

Description of the model

PRISM....

1) Station Retrieval: all station located within a prescribed maximum radius (MAXRAD) from the DEM grid cell are retrieved

The station not falling on the cell's topographic facet are omitted

If the number of remaining stations is less than a prescribed number (MINSTA), PRISM attempts to include more station by using a filtered FACET grid.

If the number of stations on the correct facet exceeds MINSTA on the first try, all station within a prescribed minimum radius (MINRAD) and on the same facet are accepted.

If no station on the same facet are available the one station that is nearest to the DEM grid cell, is selected for use in the regression calculation.

Stations located on cells that have the same orientation as the DEM grid cell but lie on the complete different fact are eliminated

Development of Precipitation –DEM Elevation Regression Functions

- Monthly and annually linear precipitation –DEM elevations regression functions are generated from the station datasets retrieved from the grid cell B1MIN
- If the slope of calculated regression line falls outside prescribed bounds (B1MIN and B1MAX) the model attempts to find an influential outlying station that is causing the unusual slope
- If an influential outlier is found, it is deleted from the regression; if it cannot be deleted the slope is set to a prescribed default value DB1
- B1MIN B1MAXand DB1 are not expressed in absolute units, are expressed as a proportion of the average precipitation of the regression dataset for each cell:

$$B_1 = \frac{b_{1\,ij}}{\overline{P}_{ij}}$$

$$\overline{P}_{ij} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} P_{k_{ij}}$$

BIMIN=0

BMAX = 3. 0 km^{-1}

Precipitation Estimation:

A monthly or annual precipitation prediction at grid cell (i ,j) is calculated as:

$$P_{ij} = b_{0_{ij}} + b_{1_{ij}} E_{ij}$$

Where b_{0ij} and b_{1ij} are the monthly regression slope and intercept

Vertical Extrapolation Adjustment for Coarse –Grid Simulation

- The precipitation –DEM elevation function is not always determined; rather is developed only on lower, gentle slope which are represented reasonably well by coarse grid DEM;

but how about the regions where the terrain alternates between flat valleys and towering, sharply defined picks?!

A temporary algorithm – adjust the precipitation estimates for locally high elevation by allowing the slope of the precipitation –DEM elevation function to be changed to a prescribed value B1EX = 2.00

Estimation of Prediction Intervals

PRISM calculates 95% prediction intervals;

- it calculates prediction intervals for the dependent variable Y;

The prediction intervals take into account both the variation in the possible location of the expected value of Y for a given X, and the variation of individuals values of Y around the expected value.

Variance:

$$s^2(Y_h) = s^2(\hat{Y}_h) + MSE = MSE\left[1 + \frac{1}{n} + \frac{\left(X_h - \hat{X}\right)^2}{\sum_{i=1}^{n}\left(X_i - \hat{X}\right)^2}\right]$$

The 95% CI:

$$\hat{Y}_h \pm t_{0.975.\,df}\, S(\hat{Y}_h)$$

Gradient

- Is a postprocessor to the PRISM estimated precipitation grids;

- Checks for discontinuities in prediction between adjacent topographic facets on cell-by-cell basis;

Model Evaluation

The performance of PRISM was compared to the geostatistical interpolation methods though application to:

a) Willamette River basin
b) Northern Oregon
c) Western United Stated

Willamette River basin

Datasets consisted of average annual precipitation for the period 1982-1988 at 52 station :

37 station operated by NWS

15 station were Soil Conservation Service SNOTEL

To this area were applied PRISM and the e geostatistical methods

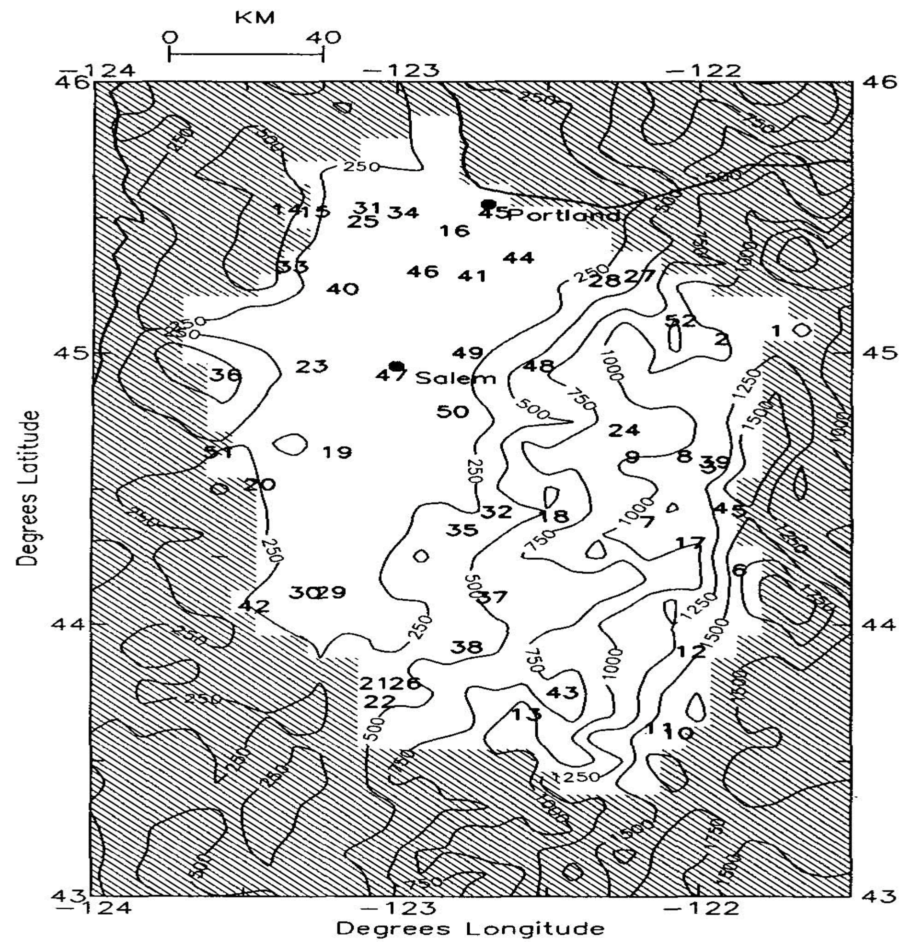


FIG. 5. The Willamette River basin modeling domain. Elevation contour interval is 250 m. Numbers 1–15 denote SNOTEL sites, and 16–52 denote NWS and cooperator stations.

Willamette...

PRISM exhibited the lowest cross-validation errors; followed by: Detrended Kriging; Cokriging; Kriging.

The bias for PRISM was 0.1 cm (-1.4, -2.0, and -5.2);

The MAE was 17 cm for PRISM (19 cm for detrended kriging; 20 cm for cokriging and 26 for kriging);

TABLE 3. Summary statistics for precipitation estimates by PRISM, kriging, detrended kriging, and cokriging in the Williamette River basin.

Method	Minimum estimate (cm)	Maximum estimate (cm)	Mean estimate (cm)	X-val* bias (cm)	Mean absolute X-val error (cm)
PRISM	93	344	157	0.1	17
Kriging	102	265	157	−5.2	26
Detrended kriging	57	259	170	−1.4	19
Cokriging	60	319	167	−2.0	20
Station precipitation values (n = 52)	99	293	164	—	—

* Cross validation.

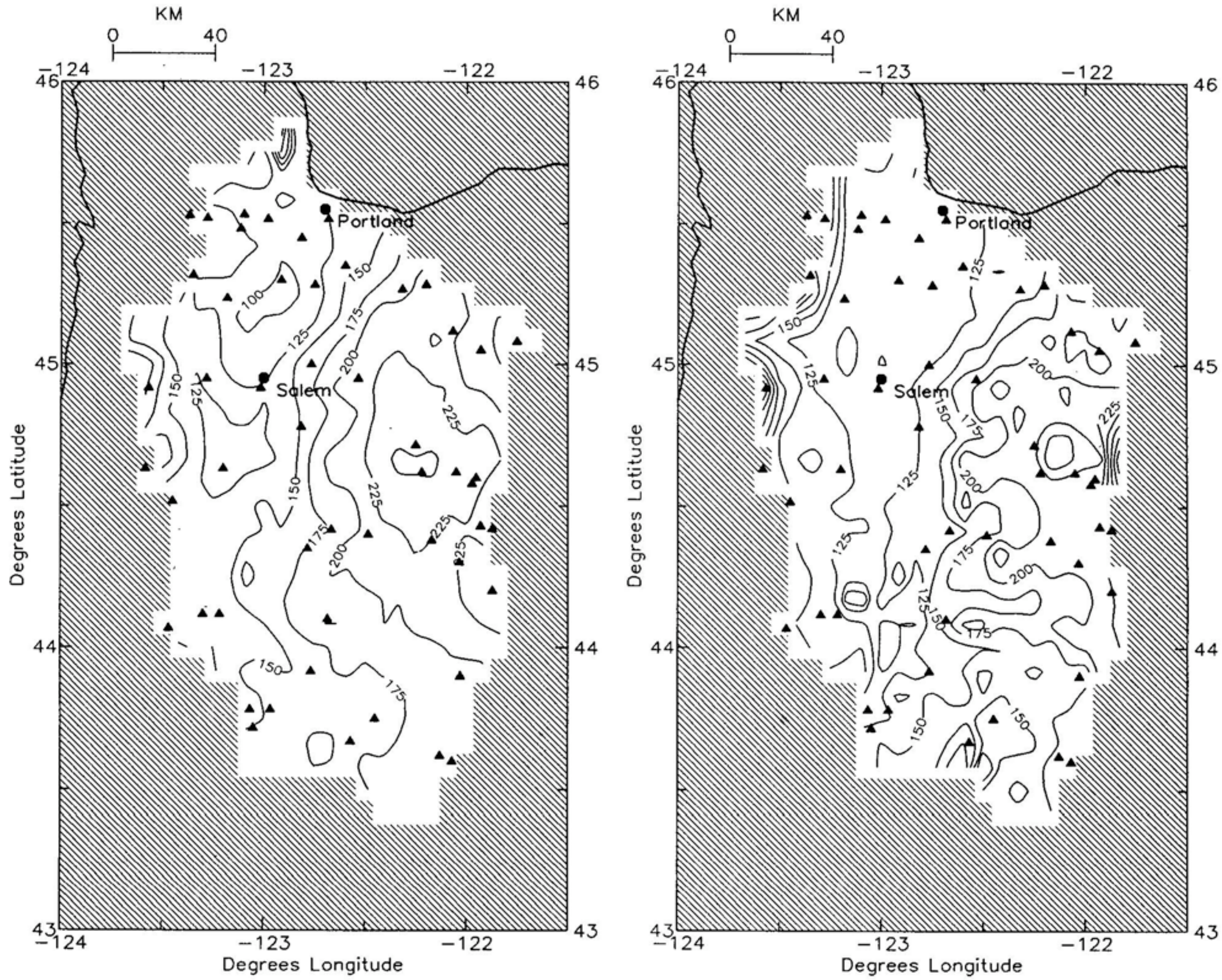


FIG. 6. Predicted annual precipitation fields for the Willamette River basin produced by (a) detrended kriging and (b) PRISM. Contour interval is 25 cm.

Willamette...

- Comparisons with the best available isohyetal map for Oregon (USDA Soil Conservation Service 1964), which provides substantial detail in mountainous areas, indicated that the precipitation field from PRISM corresponded more closely to the isohyetal map than did those from geostatistical methods.

Northern Oregon:

- PRISM was also applied to northern Oregon to assess its performance in a region that encompasses several radically different orographic regimes and incorporates extreme spatial gradients in precipitation.
- Bias = 1.3 cm > 0.1 cm (Willamette)
- MAE = 17 cm –same with Willamette

But the average estimated precipitation was 56 cm lower than in Willamette River basin

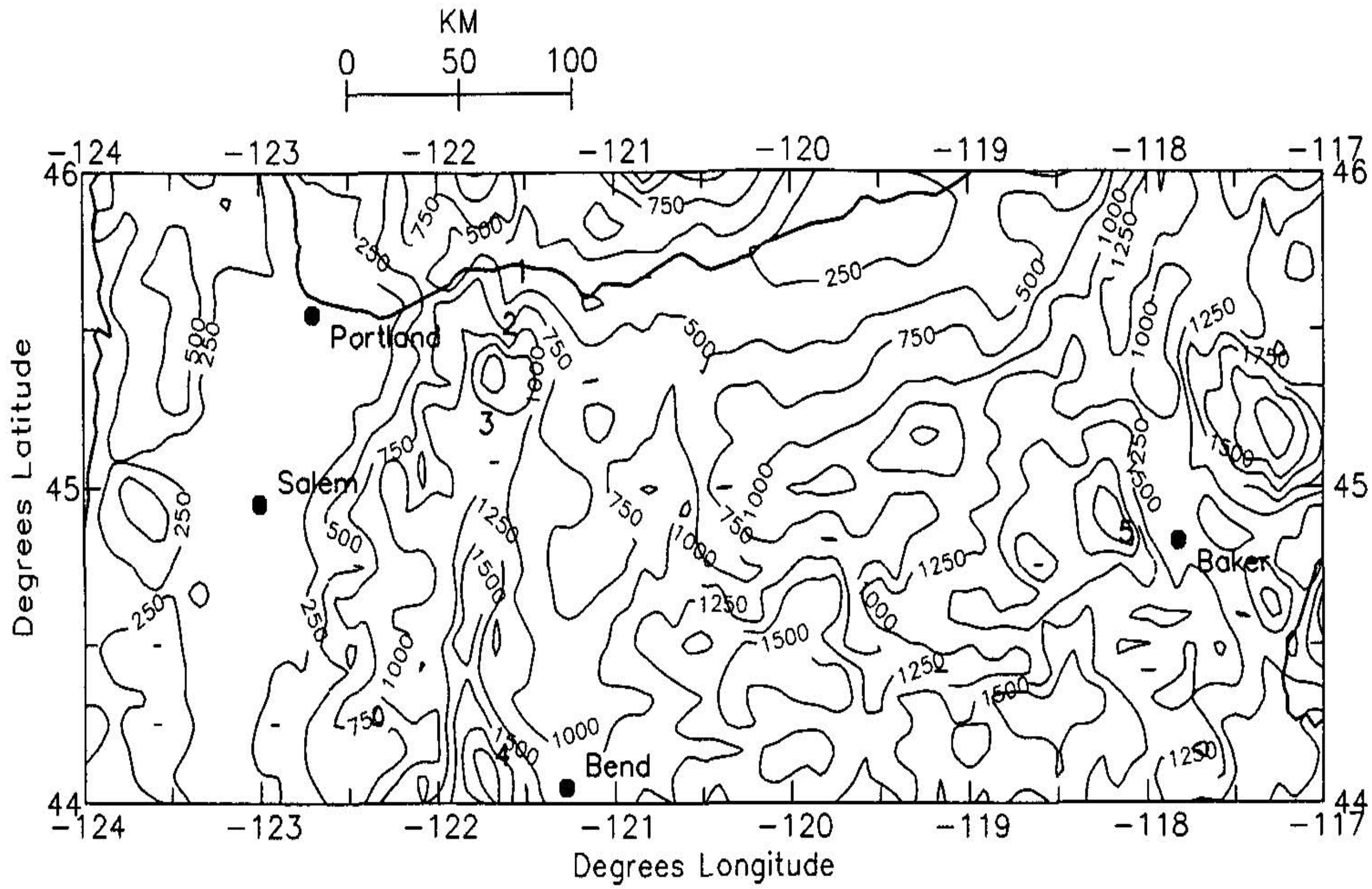


FIG. 7. Northern Oregon PRISM modeling domain. Elevation contour interval is 250 m. The numbers 1–5 indicate stations having observed annual precipitation that differed markedly from PRISM predictions in cross validation.

Contour plots of mean annual precipitation and prediction intervals

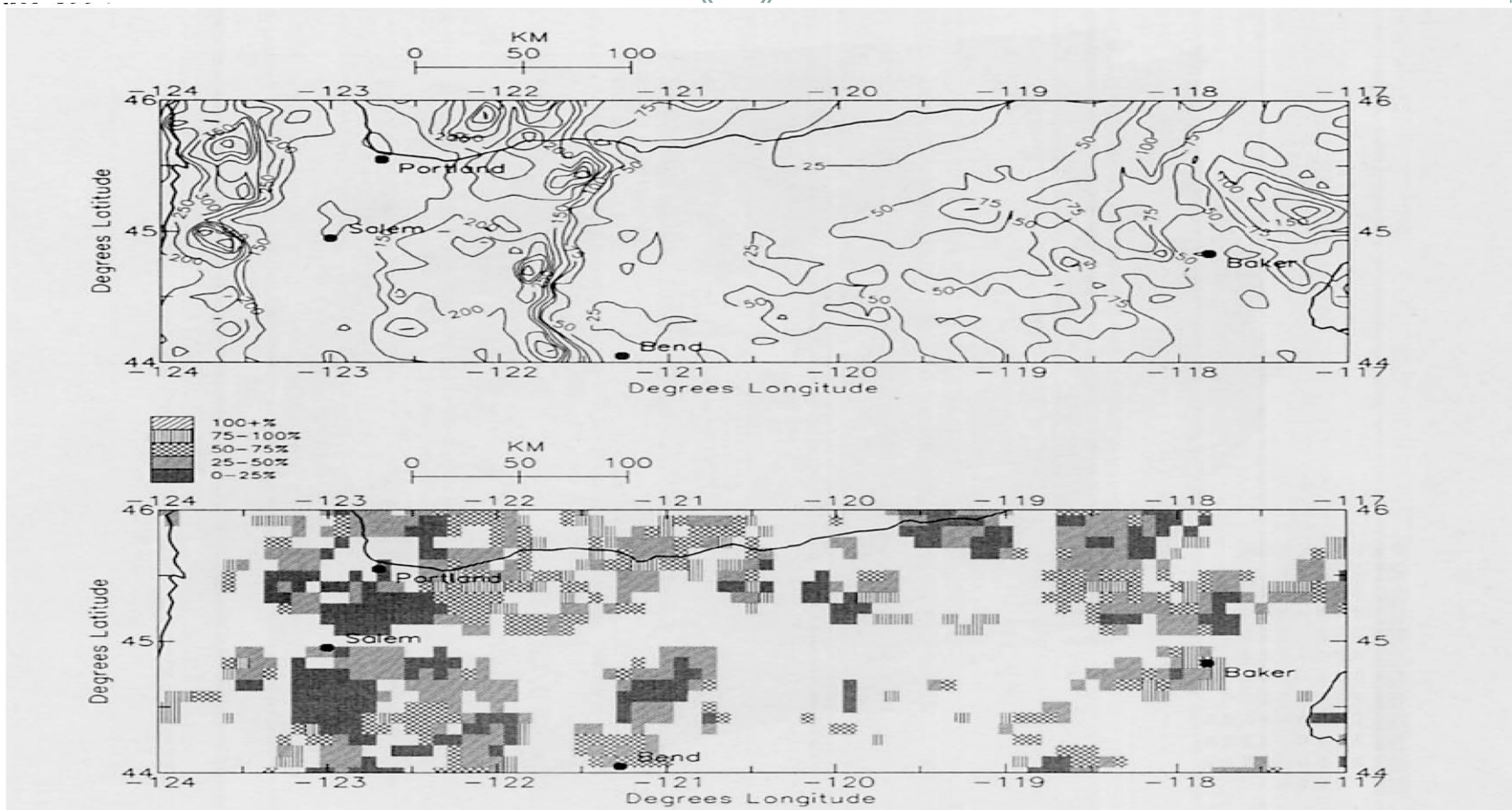


FIG. 8. Northern Oregon (a) annual precipitation and (b) 95% prediction half-intervals produced by PRISM. Contour interval for precipitation is 25 cm below 100 cm, and 50 cm above 100 cm. A prediction half-interval of 50% means there is a 95% probability that the actual precipitation is within 50% of the predicted value.

TABLE 5. Monthly and annual summary statistics for PRISM precipitation estimates for northern Oregon. Cross-validation bias and mean absolute error are expressed in centimeters and in percentage of observation. The percent value is arrived at by expressing each error as a percentage of the observation, then averaging them.

Month	Mean *P/E** regression r^2	Mean precipitation (cm)	Mean regression slope (km^{-1})	X-val** bias (% obs)	Mean absolute X-val error (% obs)
January	0.61	14	1.2	6.7	22
February	0.61	11	1.3	8.0	25
March	0.63	10	1.2	5.8	21
April	0.63	6	1.1	4.5	20
May	0.65	5	1.1	5.0	19
June	0.64	4	1.1	5.5	18
July	0.55	1	1.0	8.0	27
August	0.56	2	1.0	7.0	25
September	0.64	4	1.1	4.4	21
October	0.65	7	1.2	5.6	21
November	0.68	14	1.2	6.2	21
December	0.67	16	1.2	7.2	22
Annual	0.70	101	1.2	4.5	16

* Precipitation–DEM elevation.
** Cross validation.

Western United States

Purpose:

- To assess the ability of PRISM to maintain predictive capability over a large, complex regions;
- Bias = increased 3.5 %
- MAE increased 17%

TABLE 2. Values of input parameters for PRISM applications to the Willamette River basin, northern Oregon, and the western United States.

Parameter	Willamette River basin	Northern Oregon	Western United States
MINSTP	15 meters per grid cell	15 meters per grid cell	15 meters per grid cell
MAXRAD	7 grid cells	4 grid cells	3 grid cells
MINRAD	4 grid cells	3 grid cells	1 grid cell
MINSTA	3 stations	5 stations	6 stations
B1MIN	0.0 km	0.0 km^{-1}	0.0 km^{-1}
B1MAX	3.0 km^{-1}	3.0 km^{-1}	3.0 km^{-1}
DB1	0.8 km^{-1}	1.15 km^{-1}	0.97 km^{-1}
B1EX	2.0 km^{-1}	2.0 km^{-1}	2.0 km^{-1}
B1MAXG	N/A*	4.0 km^{-1}	4.0 km^{-1}
MINSLP	N/A	100 m	100 m

* N/A—the GRADIENT postprocessor was not needed in the Willamette Valley application.

Gridded annual precipitation produced by PRISM for the western United States

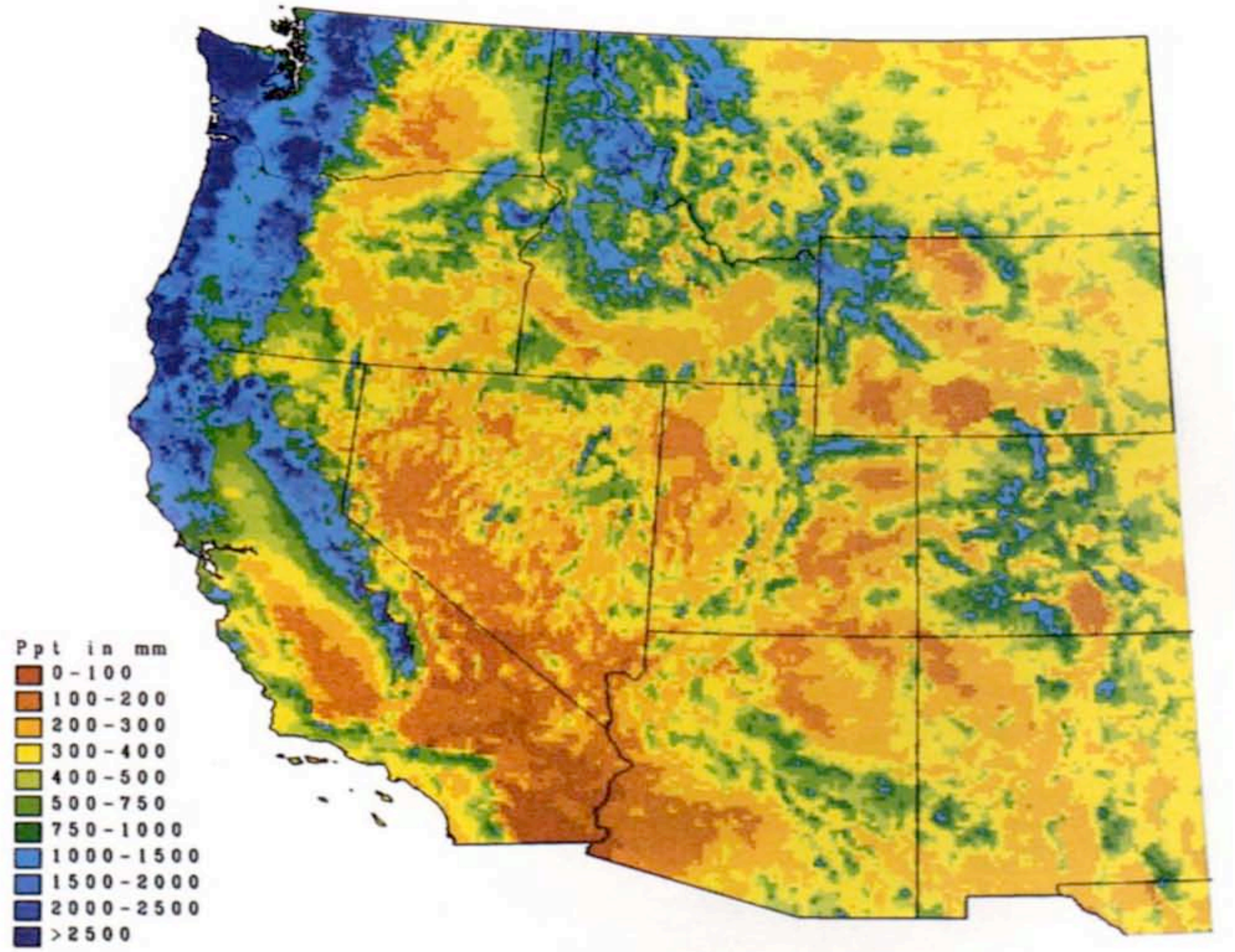


FIG. 9. Annual average precipitation for the western United States estimated by PRISM.

Summary and Future work

- PRISM brings a unique combination of climatological and statistical concepts to the mapping of orographic precipitation;
- The main focus: developing a conceptual frame work for mapping orographic precipitation in complex terrain;
- PRISM was compared to commonly used geostatistical methods –kriging, detrended kriging, and cokriging.

  all 4 were applied to Willamette River – PRISM showed superior performance

Summary and Future work...

- PRISM was applied to northern Oregon and the entire western United States – detrended kriging and cokriging could not be used;
- The ability of PRISM to maintain predictive accuracy over large areas makes it extremely useful for developing isohyetal maps;
- Is currently used to produce detailed precipitation maps of Idaho, Utah, and Nevada for the USDA Soil Conservation Service and o produce a coarser isohyetal map of the contiguous United States for the U.S Army Corps of Engineering